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Customer Influence Value and Purchase Acceleration in New Product Diffusion

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When social influence plays a key role in the diffusion of new product, the value of a customer often goes beyond her own product purchase. We posit that a customer's value (CV) comes not only from her purchase value (PV) but also from her influence value (IV) (i.e., CV = PV + IV). Therefore, a customer's value can be far greater than her purchase value if she exerts a considerable influence on others. Building on a two-segment influential–imitator asymmetric influence model, we develop a model framework to derive closed-form expressions for PV, IV, and CV by customer segment as well as time of adoption, and we examine their comparative statics with respect to the diffusion parameters. A key parameter of our model framework is the social apportioning parameter, δ, which determines the credit a customer receives by influencing other potential adopters. We develop an endogenous method for determining δ as a function of the new product diffusion parameters. Our model framework allows us to investigate how a firm might accelerate product purchases by providing introductory discount offers to a targeted group of potential adopters at product launch. We find that purchase acceleration frequently leads to a significant increase in total customer value.

Key words: new product diffusion; purchase value; influence value; purchase acceleration
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1. Introduction

New product diffusion is central to marketing. Marketing scientists have been interested in investigating how the diffusion of new product actually occurs and how firms can actively influence it. It is well known that social contagion plays a key role in how rapidly a new product diffuses. As a consequence, the value a firm derives from a specific customer goes beyond her own individual product purchase (Libai et al. 2009). To understand the precise role of social contagion, we develop a new model framework for quantifying the value of a customer in new durable product categories where social contagion is prevalent. Specifically, we posit that a customer's value (CV) is the sum of her purchase value (PV) and influence value (IV). Formally stated, we have

$$\mathrm{CV} = \mathrm{PV} + \mathrm{IV}.$$

Positive social contagion in the diffusion of new product has two distinct benefits. First, it could potentially increase the total number of adopters for a new product. Customers who otherwise would not have bought the product might now change their mind because of the positive feedback. Second, positive feedback may dramatically reduce potential adopters' timing of adoption. Uncertainty about the new product's benefits may be reduced by social contagion, and as a consequence, potential buyers may speed up their adoption process. This uncertainty reduction leads to purchase acceleration that in turn compresses the new product's life cycle. In this paper we focus exclusively on the second benefit and analyze the effect of social contagion on the timing of new product adoption and total customer value. We will characterize PV, IV, and CV in the context of the asymmetric influence model (Van den Bulte and Joshi 2007), where the total number of potential adopters is fixed but customer heterogeneity is explicitly modeled.

Following the asymmetric influence model by Van den Bulte and Joshi (2007), we divide potential adopters into two distinct segments: the *influentials* and the *imitators* segments. Each segment of potential adopters has its own respective within-segment innovation and imitation parameters, and it experiences a Bass-type diffusion process. In addition, the influentials segment can exert a cross-segment influence on the imitators segment, but not vice versa. We use this asymmetric influence model for three reasons:

1. A diffusion process that comprises a mixture of influentials and imitators appears to be consistent with several extant theories in sociology and new product diffusion research (Van den Bulte and Joshi 2007).

2. A diffusion structure with asymmetric interaction between two customer segments is not only general but also analytically tractable. It can capture the traditional symmetric-around-the-peak bell shape, asymmetric bell shapes, and a dip or "chasm" between the early and later parts of the diffusion curve (Muller and Yogev 2006, Peres et al. 2010). As a consequence, the diffusion structure has a wide applicability.

3. The proposed diffusion structure is an elegant way to incorporate heterogeneity and allows us to analytically investigate how a firm might allocate scarce promotion dollars to different customer segments at product launch in order to accelerate product purchases for profit maximization.

We use the above asymmetric influence model to characterize the dynamics of PV, IV, and CV by customer segment. These characterizations yield insights on how purchase and influence values interact over time. Our model framework shows that a natural way to increase CV is to amplify social contagion by offering introductory discounts to a targeted group of potential adopters at product launch (Marks and Kamins 1988, Van Ackere and Reyniers 1995, Jain et al. 1995, Lehmann and Esteban-Bravo 2006). Indeed, such purchase acceleration strategy is quite prevalent. For example, publishers sometimes offer introductory discounts on new college textbooks in order to accelerate their diffusion processes. Similarly, when Hasbro launched a new handheld video game called *POX* in 2001, they chose 1,600 kids to be their agents of social contagion, each armed with a backpack filled with samples of games to be distributed to their friends (Godes and Mayzlin 2009). Other examples include the widespread use of sending a new CD to a selected group of individuals for free when the CD is released.

This paper makes three contributions:

1. This paper posits that CV = PV + IV and develops a model framework for determining the value of a customer where social contagion plays an important role in new product adoption. Building on the recent asymmetric influence model by Van den Bulte and Joshi (2007), we derive closed-form expressions for PV, IV, and CV by customer segment and time of adoption. To the best of our knowledge, this is the first attempt to develop formal metrics for a firm to explicitly apportion a part of a customer's purchase value to her influencer and to formally quantify a customer's influence value over a product's life cycle.

2. We show that PV, IV, and CV always decrease in a *convex* manner with adoption time. Hence an early adopter is much more valuable than a late adopter. We show that if the influentials segment has a high cross-segment influence on the imitators segment, the CV of the latter segment drops.

3. We show how a firm can significantly increase the total CV of its entire customer base by offering introductory price discounts to a targeted group of customers at product launch. We characterize the optimal size of invited customers in terms of the level of price discount, innovation parameters, and imitation parameters. The total CV increases because the firm frequently gains more from increased IV than it loses from PV as a result of introductory discounts. As a consequence, invited early adopters become even more valuable.

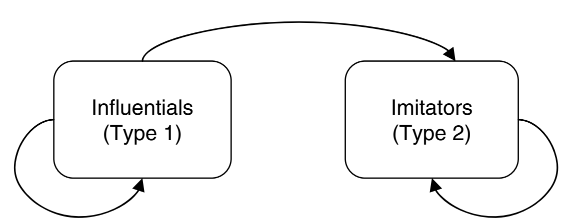
The remainder of this paper is organized as follows. Section 2 describes the proposed modeling framework. Section 3 analyzes the influence of purchase acceleration on the total customer value. Section 4 summarizes and discusses potential directions for future research. All proofs are presented in the appendix.

2. The Model

Consider a firm that introduces a new product to a fixed pool of potential adopters. We categorize potential adopters into two distinct segments: *influentials* and *imitators* (Van den Bulte and Joshi 2007). We use type 1 to denote the influentials segment and type 2 to denote the imitators segment. Each segment follows its own Bass-type diffusion process. Moreover, type 1 can exert cross-segment influence on type 2, but not vice versa. Figure 1 shows the social influence structure.

We use subscripts 1 and 2 to denote each type of potential adopters, respectively, and subscript m to denote the combined population. θ_1 is the proportion of type 1 potential adopters ($0 \le \theta_1 \le 1$), and $\theta_2 = 1 - \theta_1$ is the proportion of type 2 potential

Figure 1 Influentials and Imitators



adopters in the combined population. $f_i(t)$ and $F_i(t)$ denote the instantaneous adoption rate and cumulative adoption for type i at time t, respectively. The instantaneous adoption rates for each type and the combined population are captured by the following equations:

$$f_1(t) = (p_1 + q_1 F_1(t))(1 - F_1(t)), \tag{1}$$

$$f_2(t) = (p_2 + q_c F_1(t) + q_2 F_2(t))(1 - F_2(t)), \tag{2}$$
$$f_m(t) = \theta_1 f_1(t) + \theta_2 f_2(t).$$

Parameters p_i and q_i ($i = 1, 2$; $p_1 > 0$, $p_2 \geq 0$, and $q_1, q_2 \geq 0$) are type i's innovation and within-segment imitation parameters, respectively. As type 2's adoption behavior can also be influenced by type 1, we use q_c (≥ 0) to denote the cross-segment imitation parameter. Equation (1) suggests that an influential's likelihood of adopting at time t given that she has not adopted before t is determined by her intrinsic motivation and the within-segment social influence at that time. Equation (2) indicates that an imitator's likelihood of adopting at t given that she has not adopted before t depends on her intrinsic motivation as well as the social influence from both the influentials segment and the imitators segment at that time (Goldenberg et al. 2009).

Note that when $\theta_1 = 0$ or $\theta_1 = 1$, all potential adopters fall into a single segment, and the model is reduced to the traditional Bass diffusion model (Bass 1969). When $0 < \theta_1 < 1$ and $q_c = 0$, the two segments of potential adopters are disconnected, and each segment experiences its own Bass-type diffusion process.

If there are no prerelease purchases (i.e., $F_1(0) = F_2(0) = 0$), the cumulative adoption at t can be written as

$$F_1(t) = \frac{1 - e^{-(p_1+q_1)t}}{1 + (q_1/p_1)e^{-(p_1+q_1)t}}, \tag{3}$$

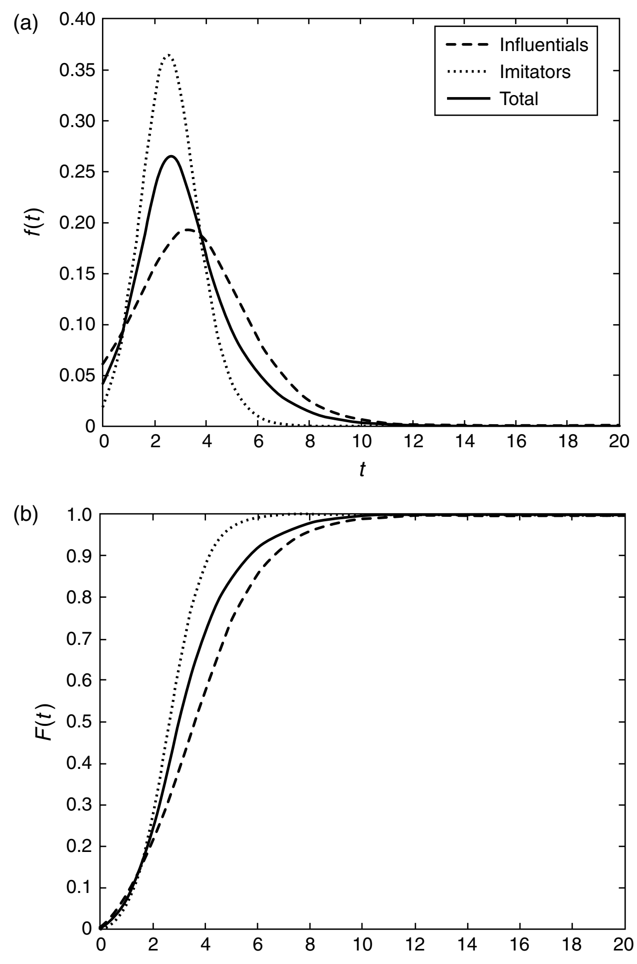
$$\begin{aligned} F_2(t) = 1 + \Bigg(& e^{-(p_2+q_2+q_c)t}\left(1 + \frac{q_1}{p_1} e^{-(p_1+q_1)t}\right)^{-q_c/q_1} \Bigg) \\ & \cdot \Bigg(q_2 \int_0^t e^{-(p_2+q_2+q_c)s}\left(1 + \frac{q_1}{p_1} e^{-(p_1+q_1)s}\right)^{-q_c/q_1} ds \\ & - \left(1 + \frac{q_1}{p_1}\right)^{-q_c/q_1} \Bigg)^{-1}, \end{aligned} \tag{4}$$
$$F_m(t) = \theta_1 F_1(t) + \theta_2 F_2(t).$$

The instantaneous and cumulative adoption functions for the asymmetric influence model are presented in Figures 2(a) and 2(b), respectively. The figures are plotted using the average parametric values of the annual data presented in Van den Bulte and Joshi (2007); i.e., $p_1 = 0.06$, $q_1 = 0.65$, $p_2 = 0.02$, $q_2 = 1.03$, $q_c = 0.62$, and $\theta_1 = 0.54$. In this average scenario, the imitators segment has a lower innovation parameter but a higher within-segment imitation parameter than the influentials segment (i.e., $p_1 > p_2$ and $q_1 < q_2$). Figure 2(a) shows that the diffusion processes are all bell shaped, each with a single peak. The instantaneous adoption rate of the combined population exhibits a clear skew to the right. In Figure 2(b), the imitators' cumulative adoption is always higher than the influentials' cumulative adoption at any time t because the former has a faster diffusion process as a result of both within- and cross-segment social contagion.

2.1. The Social Influence Chain

Our model implies a social influence chain. Consider a potential adopter, Betty, who buys at time t. Betty plays two roles in the social influence chain of the diffusion process. On one hand, she might have been

Figure 2 Instantaneous and Cumulative Adoption Functions for the Asymmetric Influence Model



Note. $p_1 = 0.06$, $q_1 = 0.65$, $p_2 = 0.02$, $q_2 = 1.02$, $q_c = 0.62$, and $\theta_1 = 0.54$.

influenced by a previous adopter, and thereby she is an *influencee*. On the other hand, she can exert social influence after her purchase and become an *influencer* of others. Consequently, she will have her own influencees. We assume that an individual can have multiple influencees but can only be influenced by at most one influencer (i.e., she may adopt the product without others' influence). As a consequence of heterogeneity and asymmetric social influence, there are three types of contagion processes: (1) influencer is type 1 and influencee is type 1, (2) influencer is type 1 and influencee is type 2, and (3) influencer is type 2 and influencee is type 2. We determine the expected number of influencees of a potential adopter who purchases the new product at time t for the three contagion scenarios as follows.

2.1.1. Type 1 Influencer and Type 1 Influencee. Let us take a close look at Betty's role as an influencer if she is a type 1 adopter who purchases the product at time t. Consider another type 1 potential adopter, Amy, who buys at time s ($s > t$). She has either been self-motivated or been influenced by a type 1 adopter who has made a purchase previously (note that type 2 adopters cannot exert influence on type 1 potential adopters). Let N_i be the size of type i potential adopters. Multiplying both sides of Equation (1) by N_1, we obtain

$$f_1(s)N_1 = p_1(1-F_1(s))N_1 + q_1F_1(s)(1-F_1(s))N_1. \quad (5)$$

From Equation (5), we know that $f_1(s)N_1$ type 1 potential adopters make their purchases at time s. Among them, $p_1(1-F_1(s))N_1$ are self-motivated, and $q_1F_1(s)(1-F_1(s))N_1$ have been influenced by other type 1 adopters. Amy, as one of the $f_1(s)N_1$ type 1 adopters at s, can either be one of the $p_1(1-F_1(s))N_1$ self-motivated adopters or be one of the $q_1F_1(s)\cdot(1-F_1(s))N_1$ influencees. Thus, the probability of her being an influencee can be written as

$$\mathbb{P}[\text{Amy is a type 1 influencee} \mid \text{Amy buys at } s] = \frac{q_1F_1(s)}{p_1+q_1F_1(s)}. \quad (6)$$

An implicit assumption in the Bass diffusion model is that "at any point in the process, all individuals who are yet to adopt have the *same* probability of adopting in a given time period, so that *differences* in individual adoption times are purely stochastic" (Chatterjee and Eliashberg 1990, p. 1058). Therefore, at any moment in time, each type 1 individual who is buying is equally likely to be an influencee of any previous type 1 buyer, and each previous type 1 buyer is equally likely to be the influencer of any type 1 individual who is buying.

At s, there are $F_1(s)N_1$ type 1 customers who have already adopted. Given that Amy is a type 1 influencee and she can only be influenced by previous type 1 adopters, each of them has equal probability of influencing her. Applying Equation (6), the probability that Amy is Betty's influencee is given by

$$\begin{aligned}\mathbb{P}[\text{Amy is Betty's influencee} \mid \text{Amy buys at } s] &= \frac{\mathbb{P}[\text{Amy is an influencee} \mid \text{Amy buys at } s]}{F_1(s)N_1} \\ &= \frac{q_1/N_1}{p_1+q_1F_1(s)}.\end{aligned}$$

As we have $f_1(s)N_1$ type 1 buyers at time s, the number of type 1 buyers at s who are influenced by Betty follows a binomial distribution with parameters $f_1(s)N_1$ and $q_1/N_1/(p_1+q_1F_1(s))$. It follows that the expected number of type 1 customers buying at s who have been influenced by Betty is

$$\mathbb{E}[\text{Number of Betty's type 1 influencees at time } s] = \frac{q_1f_1(s)}{p_1+q_1F_1(s)}.$$

Therefore, during the product life cycle, the expected total number of Betty's type 1 influencees is

$$\mathbb{E}[\text{Total number of Betty's type 1 influencees}] = \int_t^\infty \frac{q_1f_1(s)}{p_1+q_1F_1(s)}\,ds. \quad (7)$$

2.1.2. Type 1 Influencer and Type 2 Influencee. Let us return to Betty as a type 1 influencer who purchases the new product at time t. Now consider a type 2 potential adopter, Cindy, who buys at time s ($s > t$). Her adoption behavior can be motivated by herself, by a previous type 1 buyer, or by a previous type 2 buyer. In the latter two cases, Cindy's adoption is due to the social contagion process. Conditional on Cindy being influenced by a type 1 buyer, each previous type 1 buyer is equally likely to influence Cindy. Hence the probability of her being Betty's influencee is given by

$$\mathbb{P}[\text{Cindy is Betty's influencee} \mid \text{Cindy buys at } s] = \frac{q_c/N_1}{p_2+q_cF_1(s)+q_2F_2(s)}.$$

As there are $f_2(s)N_2$ type 2 buyers at time s, the number of type 2 buyers at s who are influenced by Betty follows a binomial distribution with parameters $f_2(s)N_2$ and $q_c/N_1/(p_2+q_cF_1(t)+q_2F_2(t))$. Thus the expected number of Betty's type 2 influencees at time s is

$$\mathbb{E}[\text{Number of Betty's type 2 influencees at time } s] = \frac{q_c\bar{\theta}f_2(s)}{p_2+q_cF_1(s)+q_2F_2(s)},$$

where $\bar{\theta} = N_2/N_1 = \theta_2/\theta_1$. Therefore, the expected total number of Betty's type 2 influencees over the product life cycle is

$$\mathbb{E}[\text{Total number of Betty's type 2 influencees}] = \int_t^\infty \frac{q_c \bar{\theta} f_2(s)}{p_2 + q_c F_1(s) + q_2 F_2(s)}\, ds. \tag{8}$$

Combining Equations (7) and (8), the expected total number of Betty's influencees is given by

$$\mathbb{E}[\text{Total number of Betty's influencees}] = \int_t^\infty \frac{q_1 f_1(s)}{p_1 + q_1 F_1(s)}\, ds + \int_t^\infty \frac{q_c \bar{\theta} f_2(s)}{p_2 + q_c F_1(s) + q_2 F_2(s)}\, ds. \tag{9}$$

2.1.3. Type 2 Influencer and Type 2 Influencee. Applying the same logic, the expected total number of influencees of a type 2 customer, Debby, who buys at time t can be determined as

$$\mathbb{E}[\text{Total number of Debby's influencees}] = \int_t^\infty \frac{q_2 f_2(s)}{p_2 + q_c F_1(s) + q_2 F_2(s)}\, ds. \tag{10}$$

2.2. Customer Lifetime Value

Without loss of generality, we normalize the product profit margin to one. We shall determine PV, IV, and CV by customer type and by time of adoption. Consider Betty, a type 1 adopter at time t. There is a probability $p_1/(p_1 + q_1 F_1(t))$ that she is driven by her intrinsic motivation and a probability $(q_1 F_1(t))/(p_1 + q_1 F_1(t))$ that she is influenced by others. When she is driven by her intrinsic motivation, her PV is the present value of the firm's profit derived from her (i.e., e^{-rt}, where r is the discount rate). When she is influenced by others, her PV is $(1-\delta)e^{-rt}$, where the remaining δ fraction is credited back to her influencer. Hence, Betty's PV is

$$\mathrm{PV}_1(t) = e^{-rt}\left(\frac{p_1}{p_1 + q_1 F_1(t)} + \frac{q_1 F_1(t)}{p_1 + q_1 F_1(t)}(1-\delta)\right) = e^{-rt}\left(1 - \frac{\delta q_1 F_1(t)}{p_1 + q_1 F_1(t)}\right). \tag{11}$$

As discussed above, Betty also acts as an influencer and has her own influencees. She is credited a δ fraction of the present value of the resulting profit brought in by each influencee of hers. Substituting Equations (1) and (2) into Equation (9), Betty's IV is

$$\begin{aligned}\mathrm{IV}_1(t) &= \delta \int_t^\infty e^{-rs} \frac{q_1 f_1(s)}{p_1 + q_1 F_1(s)}\, ds + \delta \int_t^\infty e^{-rs} \frac{q_c \bar{\theta} f_2(s)}{p_2 + q_c F_1(s) + q_2 F_2(s)}\, ds \\ &= \delta\left[q_1 \int_t^\infty e^{-rs}(1 - F_1(s))\, ds + q_c \bar{\theta} \int_t^\infty e^{-rs}(1 - F_2(s))\, ds\right].\end{aligned} \tag{12}$$

Now consider Debby, a type 2 adopter, who purchases at time t. She is driven by her intrinsic motivation with probability $p_2/(p_2 + q_c F_1(t) + q_2 F_2(t))$ or by social influence with probability $(q_c F_1(t) + q_2 F_2(t))/(p_2 + q_c F_1(t) + q_2 F_2(t))$. In the former, her PV is simply e^{-rt}; in the latter, her PV is $(1-\delta)e^{-rt}$. Hence, Debby's PV is

$$\begin{aligned}\mathrm{PV}_2(t) &= e^{-rt}\left(\frac{p_2}{p_2 + q_c F_1(t) + q_2 F_2(t)} + \frac{q_c F_1(t) + q_2 F_2(t)}{p_2 + q_c F_1(t) + q_2 F_2(t)}(1-\delta)\right) \\ &= e^{-rt}\left(1 - \frac{\delta(q_c F_1(t) + q_2 F_2(t))}{p_2 + q_c F_1(t) + q_2 F_2(t)}\right).\end{aligned} \tag{13}$$

After her purchase, Debby becomes an influencer of future buyers. Note that a type 1 adopter can influence a type 2 potential adopter, but not vice versa. As a result, Debby, a type 2 adopter, can only have type 2 influencees. She is credited a δ fraction of the present value from each of her influencees. Applying Equations (2) and (10), Debby's IV is

$$\mathrm{IV}_2(t) = \delta \int_t^\infty e^{-rs} \frac{q_2 f_2(s)}{p_2 + q_c F_1(s) + q_2 F_2(s)}\, ds = \delta q_2 \int_t^\infty e^{-rs}(1 - F_2(s))\, ds. \tag{14}$$

For each type of customer, the CV is simply the sum of his or her PV and IV as follows:

$$\mathrm{CV}_i(t) = \mathrm{PV}_i(t) + \mathrm{IV}_i(t) \quad (i = 1, 2).$$

Proposition 1 characterizes a customer's PV, IV, and CV by customer type and time of adoption.

Proposition 1. *The* PV, IV, *and* CV *of the influentials and imitators segments at adoption time t are characterized in Table* 1.

Note that Table 1 explicitly decomposes the firm's total profit into individual customer values by customer type and time of adoption. Also, the total CV from the entire customer base equals the firm's total profit:

$$\sum_{i=1}^{2} \theta_i \int_{t=0}^\infty \mathrm{CV}_i(t) f_i(t)\, dt = \sum_{i=1}^{2} \theta_i \int_{t=0}^\infty e^{-rt} f_i(t)\, dt. \tag{15}$$

Table 1 suggests that it may be possible to increase the firm's profit by trading off purchase value with influence value. We shall show how this can be accomplished by the so-called "purchase acceleration" or "product life cycle compression" in §3.

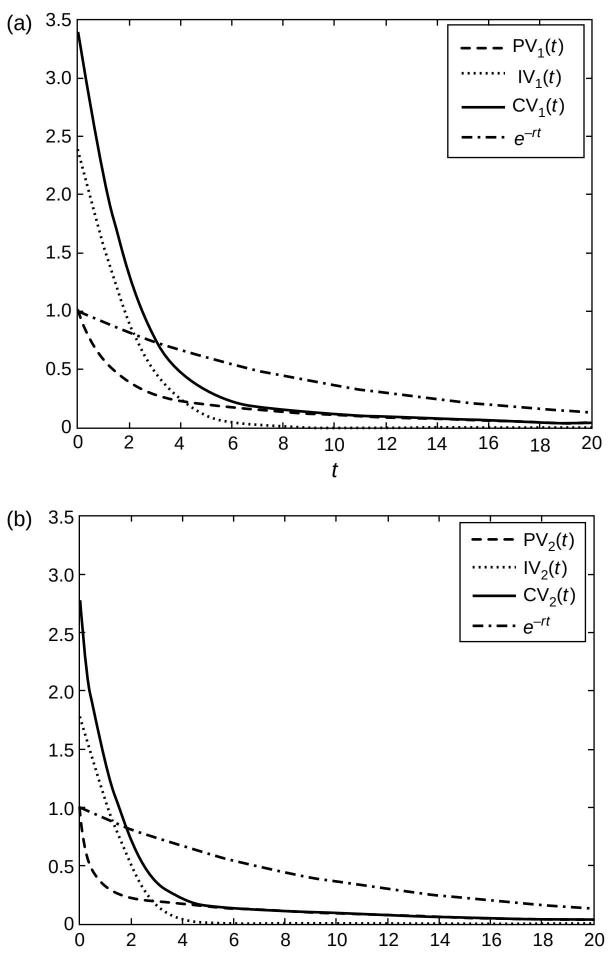
Figures 3(a) and 3(b) plot the PV, IV, and CV for each customer segment by time of adoption (with customer value computed by the traditional method

Table 1 Customer PV, IV, and CV in New Product Diffusion

PV(t)	
Type 1	$e^{-rt}\left(1-\frac{\delta q_1 F_1(t)}{p_1+q_1F_1(t)}\right)$
Type 2	$e^{-rt}\left(1-\frac{\delta(q_cF_1(t)+q_2F_2(t))}{p_2+q_cF_1(t)+q_2F_2(t)}\right)$
IV(t)	
Type 1	$\delta\left(q_1\int_t^{\infty}e^{-rs}(1-F_1(s))\,ds+q_c\bar{\theta}\int_t^{\infty}e^{-rs}(1-F_2(s))\,ds\right)$
Type 2	$\delta q_2\int_t^{\infty}e^{-rs}(1-F_2(s))\,ds$
CV(t)	
Type 1	$e^{-rt}\left(1-\frac{\delta q_1 F_1(t)}{p_1+q_1F_1(t)}\right)+\delta\left(q_1\int_t^{\infty}e^{-rs}(1-F_1(s))\,ds+q_c\bar{\theta}\int_t^{\infty}e^{-rs}(1-F_2(s))\,ds\right)$
Type 2	$e^{-rt}\left(1-\frac{\delta(q_cF_1(t)+q_2F_2(t))}{p_2+q_cF_1(t)+q_2F_2(t)}\right)+\delta q_2\int_t^{\infty}e^{-rs}(1-F_2(s))\,ds$

Notes. $F_1(t)$ and $F_2(t)$ are defined in Equations (3) and (4), respectively. General solutions are provided in the appendix.

Figure 3 PV, IV, and CV for Influentials and Imitators



Note. $p_1 = 0.06$, $q_1 = 0.65$, $p_2 = 0.02$, $q_2 = 1.02$, $q_c = 0.62$, $\theta_1 = 0.54$, $r = 0.1$, and $\delta = 0.75$.

added for ease of comparison). The figures use the average parametric values of the annual data presented in Van den Bulte and Joshi (2007); i.e., $p_1 = 0.06$, $q_1 = 0.65$, $p_2 = 0.02$, $q_2 = 1.02$, $q_c = 0.62$, and $\theta_1 = 0.54$. The influencer's social apportioning parameter, $\delta = 0.75$, is endogenously determined by a method to be discussed in §2.3. The yearly discount rate r is set to 0.1. We find that the PV, IV, and CV of both segments decrease over time. The high CV of early adopters (both influentials and imitators segments) is due to both their higher PV and their greater post-purchase social influence on later adopters. Comparing Figures 3(a) and 3(b), we find that the CV of the influentials segment is greater than that of the imitators segment in the early adoption period because of the significant cross-segment influence of the influentials on the imitators. In both figures, the CV of early adopters as computed under the proposed model framework is much higher than the customer value as computed by the traditional method. However, the former also decreases at a much faster rate over time, reversing the relationship for late adopters.

Proposition 2 establishes how a customer's purchase value, influence value, and customer value vary with the time of adoption.

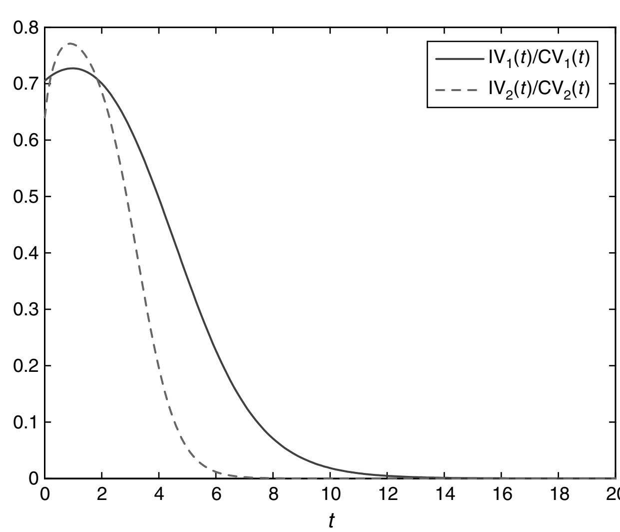
Proposition 2. PV$_i(t)$, IV$_i(t)$, *and* CV$_i(t)$ $(i = 1, 2)$ *are all decreasing and convex in* t.

When a potential adopter delays her purchase, her PV decreases because of time discounting. Her IV decreases too because the pool of potential adopters shrinks while the pool of adopters grows. As a consequence, her CV decreases rapidly as she delays her adoption of the new product. Interestingly, Proposition 2 states that the rate of decrease in PV, IV, or CV becomes smaller over time. Hence, the incremental benefit of purchase acceleration becomes larger as the product life cycle is shortened. Note that this same result continues to hold even if the product profit margin decreases (perhaps resulting from lower prices) over time. If the product margin declines over time, early adopters become more important because they will have even higher PV and IV compared with later adopters.

Figure 4 illustrates the proportion of IV as a component of CV over time. For both influentials and imitators segments, IV accounts for a greater part of CV for early adopters than for late adopters. Ho et al. (2002) show that it may be optimal to preproduce the new product before launching it in the market so as to avoid losing early adopters. Our result provides a rationale for this result from a customer value perspective. Given the significant influence of early adopters, the firm might wish to increase postpurchase customer service early in the product life cycle because such a strategy would increase customer satisfaction, which in turn will accelerate the positive social contagion process.

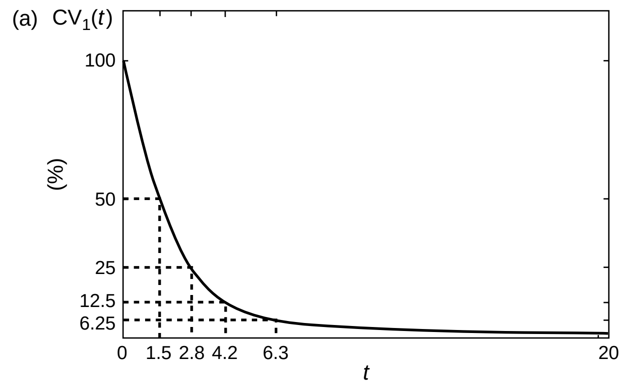
It is worthwhile to determine the half-life of CV (i.e., the time it takes for CV to decrease by half). Figures 5(a) and 5(b) show the half-life of CV for the influentials and imitators segments, respectively. Note that for both segments, the half-life of CV is quite short (e.g., the total time it takes for CV to drop by half twice is less than three years for both segments). In addition, we observe that the half-life of the imitators segment is shorter than that of the influentials segment because of the faster diffusion process in the former (note that in Figure 2(b), the cumulative adoption for the imitators segment is always higher).

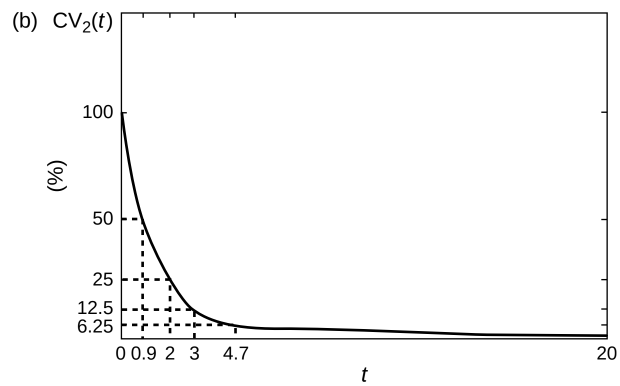
Figure 4 **Proportion of IV for Influentials and Imitators**



Note. $p_1 = 0.06$, $q_1 = 0.65$, $p_2 = 0.02$, $q_2 = 1.02$, $q_c = 0.62$, $\theta_1 = 0.54$, $r = 0.1$, and $\delta = 0.75$.

Figure 5 **Half-Life of CV for Influentials and Imitators**





Note. $p_1 = 0.06$, $q_1 = 0.65$, $p_2 = 0.02$, $q_2 = 1.02$, $q_c = 0.62$, $\theta_1 = 0.54$, $r = 0.1$, and $\delta = 0.75$.

2.3. An Endogenous Method of Determining δ

The modeling framework assumes that for each new product adoption that is driven by social influence, a δ fraction of the profit margin derived from each influencee is being credited to her influencer. Hence, δ plays a central role in determining a customer's PV, IV, and CV. It is therefore natural to ask how δ can be determined theoretically and empirically. In this subsection, we describe an endogenous method for determining δ.

Our asymmetric influence model consists of a set of innovation parameters, $\mathbf{p} = (p_1, p_2)$, and imitation parameters, $\mathbf{q} = (q_1, q_2, q_c)$. Let the total customer value of the combined population be $\mathrm{CV}_m(\mathbf{p}, \mathbf{q})$. We then create a hypothetical population by suppressing social influence (i.e., by setting imitation parameters to zero; $\mathbf{q} = (q_1, q_2, q_c) = \mathbf{0}$). Let us denote the total customer value of this hypothetical population by $\mathrm{CV}_m(\mathbf{p}, \mathbf{0})$. Clearly, $\mathrm{CV}_m(\mathbf{p}, \mathbf{q}) > \mathrm{CV}_m(\mathbf{p}, \mathbf{0})$, and the incremental value as a result of the social contagion process equals $\mathrm{CV}_m(\mathbf{p}, \mathbf{q}) - \mathrm{CV}_m(\mathbf{p}, \mathbf{0})$. Let $\mathrm{IV}_m(\mathbf{p}, \mathbf{q})$ be the total influence value of the combined population. From Equations (12) and (14), we have

$$\mathrm{IV}_m(\mathbf{p}, \mathbf{q}) = \sum_{i=1}^{2} \theta_i \int_{t=0}^{\infty} \mathrm{IV}_i(t) f_i(t)\, dt.$$

Note that $\text{IV}_m(\mathbf{p},\mathbf{q}) = \text{CV}_m(\mathbf{p},\mathbf{q}) - \text{CV}_m(\mathbf{p},\mathbf{0})$. Therefore, one can endogenously determine δ by the following equation:

$$\sum_{i=1}^{2}\theta_i\int_{t=0}^{\infty}\text{IV}_i(t)f_i(t)\,dt = \text{CV}_m(\mathbf{p},\mathbf{q}) - \text{CV}_m(\mathbf{p},\mathbf{0}). \tag{16}$$

Applying the closed-form expressions of PV, IV, and CV defined in §2.2, Equation (16) leads to the following proposition.

Proposition 3. *The social apportioning parameter $0 \le \delta \le 1$ is uniquely and endogenously determined by the following equation*:

$$\begin{aligned}\frac{\theta_1 p_1}{p_1+r}+\frac{\theta_2 p_2}{p_2+r} &= \theta_1\int_{t=0}^{\infty}e^{-rt}(1-F_1(t))(p_1+(1-\delta)q_1F_1(t))\,dt\\ &\quad+\theta_2\int_{t=0}^{\infty}e^{-rt}(1-F_2(t))(p_2+(1-\delta)\\ &\quad\cdot\big(q_cF_1(t)+q_2F_2(t))\big)\,dt. \end{aligned}\tag{17}$$

Note that given a set of innovation and imitation parameters $(\mathbf{p},\mathbf{q})$, we can uniquely determine δ. In the special case of simple Bass diffusion model (i.e., $\theta_2 = 0$), one can derive a closed-form expression for δ as follows:

$$\begin{aligned}\delta = \bigg[&-\frac{1}{b+ab(1+b)}+\frac{a+1}{a+2}\,{}_2F_1(1,a+2;a+3;-b)\\ &+\frac{1-ab}{(a+1)b}\,{}_2F_1(1,a+1;a+2;-b)\bigg]\\ \cdot\bigg[&\frac{a+1}{a+2}\,{}_2F_1(1,a+2;a+3;-b)\\ &-\frac{a}{a+1}\,{}_2F_1(1,a+1;a+2;-b)\bigg]^{-1},\end{aligned}$$

where $a = r/(p_1+q_1)$, $b = q_1/p_1$, and ${}_2F_1(x_1,x_2;y;z)$ is the Gaussian hypergeometric function.

In general, there is no closed-form expression for δ. We numerically compute δ under various parametric conditions. Table 2 summarizes the result. We use the average parametric values of the annual data in Van den Bulte and Joshi (2007) as the base case, and we systematically vary each diffusion parameter to study the sensitivity of δ with respect to it. Under this average diffusion scenario, we find that δ decreases in p_1, p_2, and q_1, and it increases in q_2 and q_c. The results imply that each influencer should be given more credit when the imitators segment's within-segment imitation parameter or the cross-segment imitation parameter increases, but less credit when either segment's innovation parameter or the influentials segment's within-segment imitation parameter increases.

Table 2 Sensitivity Analysis of δ

p_1	0.04	0.06	0.08	0.10	0.12	0.14
δ	0.79	0.75	0.72	0.70	0.68	0.67
p_2	0.002	0.005	0.010	0.020	0.030	0.040
δ	0.82	0.81	0.78	0.74	0.70	0.67
q_1	0.2	0.4	0.6	0.8	1.0	1.2
δ	0.78	0.76	0.75	0.74	0.74	0.74
q_2	0.1	0.3	0.5	0.7	1.0	2.0
δ	0.74	0.74	0.75	0.75	0.75	0.75
q_c	0.1	0.3	0.5	0.8	1.0	2.0
δ	0.74	0.75	0.75	0.75	0.75	0.75

Note. $p_1 = 0.06$, $q_1 = 0.65$, $p_2 = 0.02$, $q_2 = 1.02$, $q_c = 0.62$, $\theta_1 = 0.54$, and $r = 0.1$.

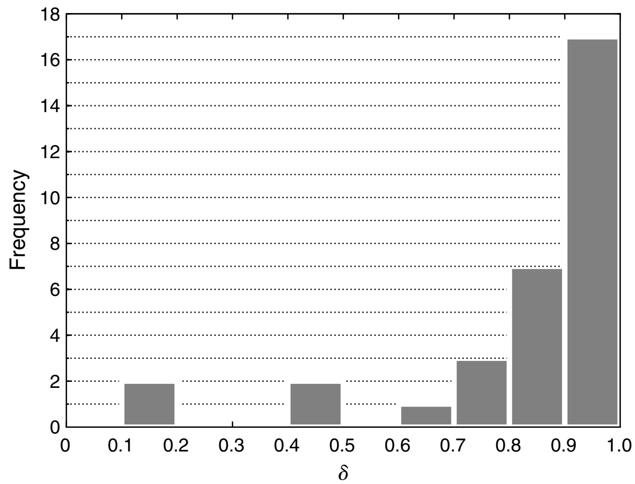
To gain further managerial insight, we examine the distribution of δ for 32 products from Van den Bulte and Joshi (2007).[1] Figure 6 shows the frequency plot of δ for this sample of 32 products. Note that there are 17 products with δ greater than 0.9 and 25 products with δ greater than 0.8. The very high δ value in a significant majority of these products indicates that a large fraction of the profit generated by each product adoption should be credited to the corresponding influencer. This result highlights the importance of social contagion and the significance of customer influence value in most new product diffusions. We also observe that a low value of δ is typically associated with a very small cross-segment imitation parameter (e.g., the three products that have the smallest δ values all have their cross-segment imitation parameters less than $1.50 \cdot 10^{-5}$). In summary, as δ does vary across products, it is important for the firm to understand what its new product's δ value is so that it can correctly determine the CV for each of its customers and properly allocate its limited marketing resources.

2.4. Comparative Statics

In this section, we study how the PV, IV, and CV of the influentials and imitators segments vary with innovation and imitation (both within-segment and cross-segment) parameters. For each dependent variable of interest, we break down the comparative statics into two parts: the direct effect and indirect effect. The direct effect refers to the effect a diffusion parameter directly has on the dependent variable of interest, whereas the indirect effect refers to the effect a diffusion parameter indirectly has on the dependent variable of interest "through" the social apportioning parameter δ, which is endogenously determined in §2.3. The total effect of a diffusion parameter on the dependent variable of interest is defined as the sum

[1] We excluded one product whose data are neither weekly nor yearly. Parametric values estimated to be 0.000 are set to 0.0005 for this investigation.

Figure 6 Frequency Analysis of δ in 32 Products



of the direct effect and indirect effect. Stated formally, we have

$$\underbrace{\frac{\partial Y(x,\delta(x))}{\partial x}}_{\text{Total effect}} = \underbrace{\frac{\partial Y(x,\delta)}{\partial x}}_{\text{Direct effect}} + \underbrace{\frac{\partial \delta(x)}{\partial x}\cdot\frac{\partial Y(x,\delta)}{\partial \delta}}_{\text{Indirect effect}},$$

where $Y \in \{\mathrm{PV}_1, \mathrm{PV}_2, \mathrm{IV}_1, \mathrm{IV}_2, \mathrm{CV}_1, \mathrm{CV}_2\}$ is a dependent variable of interest, and $x \in \{p_1, p_2, q_1, q_2, q_c\}$ is a diffusion parameter.

We provide comparative statics results for both the direct and total effects for two reasons. First, the direct effect varies in a more straightforward manner with respect to each diffusion parameter and hence gives us a sharper view as to the likely directional change of the dependent variable of interest. Second, because the indirect effect represents the effect a diffusion parameter has "through" the social apportioning parameter, which typically varies very little locally, the total effect frequently has the same directional change as the direct effect. This decomposition analysis, therefore, helps us to understand the incremental effect each parameter "indirectly" has on the dependent variable of interest via the social apportioning parameter.

2.4.1. Comparative Statics—Direct Effects.

Proposition 4. *The direct effects with respect to innovation parameters (p_1 and p_2) are*[2]

(a) $\mathrm{PV}_1(t)$ *increases with* p_1; $\mathrm{PV}_2(t)$, $\mathrm{IV}_1(t)$, $\mathrm{IV}_2(t)$, *and* $\mathrm{CV}_2(t)$ *all decrease with* p_1; *and* $\mathrm{CV}_1(t)$ *decreases with* p_1 *for all* $t < t_1$, *where*

$$t_1 = \min\left\{t\colon e^{-rt}\frac{(1-(p_1t+q_1t+1)e^{-(p_1+q_1)t})}{(p_1+q_1)^2} = \int_t^\infty e^{-rs}\frac{\partial F_1(s)}{\partial p_1}\,ds + \frac{q_c\bar{\theta}}{q_1}\int_t^\infty e^{-rs}\frac{\partial F_2(s)}{\partial p_1}\,ds\right\}.$$

(b) $\mathrm{PV}_1(t)$ *is independent of* p_2; $\mathrm{PV}_2(t)$ *increases with* p_2; $\mathrm{IV}_1(t)$, $\mathrm{IV}_2(t)$, *and* $\mathrm{CV}_1(t)$ *all decrease with* p_2; *and* $\mathrm{CV}_2(t)$ *decreases with* p_2 *for all* $t < t_2$, *where*

$$t_2 = \min\left\{t\colon \frac{e^{-rt}((q_c/q_2)F_1(t)+F_2(t)-p_2(\partial F_2(t)/\partial p_2))}{(p_2+q_cF_1(t)+q_2F_2(t))^2} = \int_t^\infty e^{-rs}\frac{\partial F_2(s)}{\partial p_2}\,ds\right\}.$$

Proposition 4(a) suggests that if the influentials segment has a higher innovation parameter, the segment will have a higher PV. This is so because a high innovation parameter means that many potential adopters purchase the new product on their own without being influenced by others. Also, the influentials segment's IV decreases as its innovation parameter increases for the same reason. The net effect of the innovation parameter p_1 on the influentials segment's CV, as a consequence, depends on the relative size of increase in PV and decrease in IV. Because early adopters are more valuable than late adopters (see Proposition 2), the loss from IV surpasses the gain from PV during the early period of the adoption process. As a result, the CV of the influentials segment's adopters who buy before t_1 decreases as its innovation parameter increases. We also observe that the PV, IV, and CV of the imitators segment decrease as the influentials segment's innovation parameter increases, suggesting that faster diffusion as a result of higher innovation in the influentials segment makes the imitators segment more likely to adopt via cross-segment social contagion rather than via the imitators segment's own innovation or within-segment social influence.

Proposition 4(b) states that the PV of the imitators segment increases as the segment's innovation parameter increases. The IV for the imitators segment, however, decreases with its innovation parameter. As a result, the CV of early adopters who adopt before t_2 in the imitators segment decreases with its innovation parameter, because the loss in IV is greater than the gain in PV for these adopters. Because imitators cannot influence influentials, the PV of the influentials segment does not depend on the imitators segment's innovation parameter. The IV and thus the CV of the influentials segment decrease as the imitators segment's innovation parameter increases, as the relative importance of cross-segment influence becomes smaller.

Proposition 5. *The direct effects with respect to imitation parameters (q_1, q_c, and q_2) are*

(a) $\mathrm{PV}_1(t)$, $\mathrm{PV}_2(t)$, $\mathrm{IV}_2(t)$, *and* $\mathrm{CV}_2(t)$ *all decrease with* q_1; *and* $\mathrm{IV}_1(t)$ *and* $\mathrm{CV}_1(t)$ *both decrease with* q_1 *for* $t > t_3$, *where* t_3 *is defined as*[3]

$$t_3 = \max\left\{t\colon \int_t^\infty e^{-rs}\left(1-F_1(s)-q_1\frac{\partial F_1(s)}{\partial q_1}-q_c\bar{\theta}\frac{\partial F_2(s)}{\partial q_1}\right)ds = 0\right\}.$$

[2] $t_i = \infty (i = 1, 2)$ if no such minimum exists.

[3] $t_3 = 0$ if no such maximum exists.

(b) $\mathrm{PV}_1(t)$ is independent of q_c; $\mathrm{PV}_2(t)$, $\mathrm{IV}_2(t)$, and $\mathrm{CV}_2(t)$ all decrease with q_c; and $\mathrm{IV}_1(t)$ and $\mathrm{CV}_1(t)$ both decrease with q_c if

$$\int_t^\infty e^{-rs}\left(1-F_2(s)-q_c\frac{\partial F_2(s)}{\partial q_c}\right)ds<0.$$

(c) $\mathrm{PV}_1(t)$ is independent of q_2; $\mathrm{PV}_2(t)$, $\mathrm{IV}_1(t)$, and $\mathrm{CV}_1(t)$ all decrease with q_2; and $\mathrm{IV}_2(t)$ and $\mathrm{CV}_2(t)$ both decrease with q_2 if

$$\int_t^\infty e^{-rs}\left(1-F_2(s)-q_2\frac{\partial F_2(s)}{\partial q_2}\right)ds<0.$$

Proposition 5(a) suggests that the PV for both segments decreases as the influentials segment's within-segment imitation parameter (q_1) increases because more potential adopters purchase the new product as a result of either within- or cross-segment social influence. Interestingly, the IV for the imitators segment decreases as q_1 increases as more purchase value in the imitators segment is being credited to the influentials segment. CV (=PV + IV) for the imitators segment, as a result, decreases. The IV and CV for the influentials segment decrease as q_1 increases when the timing of adoption is sufficiently large. This is because conditional on a large time of adoption, few potential adopters remain. As a result, the IV and hence CV decrease.

Proposition 5(b) states that the PV, IV, and CV for the imitators segment all decrease as the cross-segment imitation parameter (q_c) increases. This is so because a higher cross-segment imitation parameter indicates that imitators are more likely to adopt as a result of cross-segment social influence rather than within-segment innovation or social influence. The PV for the influentials segment is independent of q_c. Both the IV and CV of the influentials segment decrease with q_c if the time of adoption is large enough or the pool of remaining potential adopters in the imitators segment is small enough.

Proposition 5(c) states that PV for the influentials segment is independent of the imitators segment's within-segment imitation parameter (q_2). The IV and CV for the influentials segment decrease as q_2 increases because less purchase value of the imitators segment is being credited to the influentials segment. Put differently, a customer in the imitators segment is more likely to be influenced by another imitator than by an influential. Clearly, the PV for the imitators segment decreases as q_2 increases. The IV and CV for the imitators segment decrease with q_2 if the time of adoption is large enough or the pool of remaining potential adopters in the imitators segment is small enough.

2.4.2. Comparative Statics—Total Effects. The total effect of each diffusion parameter on a dependent variable of interest (i.e., PV, IV, or CV) is not analytically tractable. We therefore conduct an extensive numerical simulation using the estimated parametric values given in Van den Bulte and Joshi (2007). We focus on new products whose cross-segment imitation parameter q_c is greater than or equal to 0.001 so that there exists a significant cross-segment social influence by the influentials segment. We systematically vary each diffusion parameter by ±20% from its estimated value and evaluated PV, IV, and CV at 20 points in time $t(i)$ ($i=0$, 0.05, 0.10, ..., 0.90, 0.95), where $t(i)$ refers to the time by which i proportion of the combined population have adopted the new product. We label the total effect with respect to a diffusion parameter as "conclusive" if it has the same directional change for *all* new products at *all* 20 points in time.

Table 3 Comparative Statics—Direct Effects and Total Effects

	$\mathrm{PV}_1(t)$	$\mathrm{IV}_1(t)$	$\mathrm{CV}_1(t)$	$\mathrm{PV}_2(t)$	$\mathrm{IV}_2(t)$	$\mathrm{CV}_2(t)$
Direct effects						
p_1	↑	↓	↓ for $t<t_1$	↓	↓	↓
p_2	0[a]	↓	↓	↑	↓	↓ for $t<t_2$
q_1	↓	↓ for $t>t_3$	↓ for $t>t_3$	↓	↓	↓
q_2	0	↓	↓	↓	↓ if C1 holds[b]	↓ if C1 holds
q_c	0	↓ if C2 holds[c]	↓ if C2 holds	↓	↓	↓
Total effects						
p_1	—[d]	—	—	—	—	—
p_2	↑	↓	—	↑	↓	—
q_1	↓	—	—	—	↓	—
q_2	—	—	—	↓	—	—
q_c	—	—	—	↓	↓	↓

[a]The value in the column is independent of the corresponding diffusion parameter.
[b]C1: $\int_{s=t}^{\infty} e^{-rs}(1-F_2(s)-q_2(\partial F_2(s)/\partial q_2))\,ds<0$.
[c]C2: $\int_{s=t}^{\infty} e^{-rs}(1-F_2(s)-q_c(\partial F_2(s)/\partial q_c))\,ds<0$.
[d]The comparative statics is inconclusive.

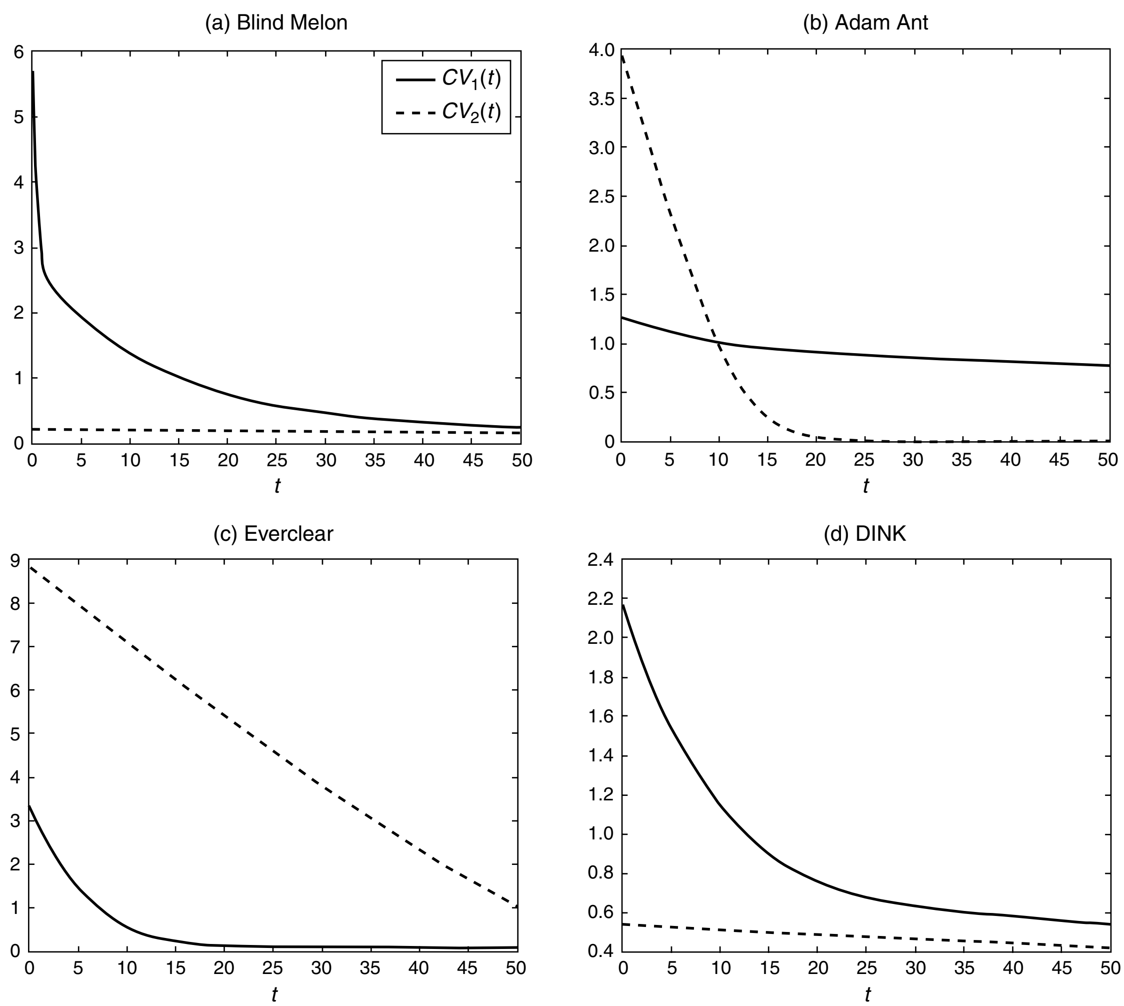
Table 3 summarizes the comparative statics results for both the direct and total effects. The total effects have 10 conclusive cases, 9 of them having the same directional change as those in the direct effects. The only conclusive case of the total effects that does not coincide with that of the direct effects is the comparative statics result on the influentials segment's PV with respect to the imitators segment's innovation parameter p_2. This is because the imitators segment's innovation parameter imposes no direct effect on the influentials segment's PV, but it exerts indirect influence on the influentials segment's PV via the social apportioning parameter.

It is worth highlighting several conclusive results and discussing their implications. First, the PV, IV, and CV of the imitators segment tend to decrease as the cross-segment imitation parameter (q_c) increases. As a result, the imitators segment may become less valuable compared with the influentials segment. Therefore, the firm may wish to gradually divert resources away from the imitators segment as q_c increases. Second, the PV for both segments tends to increase, and the IV for both segments tends to decrease as the imitators segment's innovation parameter (p_2) increases. Put differently, p_2 reduces the relative importance of social contagion. Therefore, early adopters may be less valuable when p_2 is high.

2.5. Empirical Analysis

In this subsection, we investigate the dynamics of customer value using the diffusion parameters of four CDs from Van den Bulte and Joshi (2007). Because people normally do not buy two identical CDs for themselves, music CDs are like durable goods. (Van den Bulte and Joshi 2007, p. 411) show that these CDs capture four typical diffusion paths: (1) "a rather smooth decline" for Blind Melon, (2) an "early dip followed

Figure 7 Customer Value of Four CDs



Notes. In (a), $p_1 = 0.21$, $q_1 = 3.29$, $p_2 = q_2 = 0$, $q_c = 0.073$, $\theta_1 = 0.24$, $r = 0.005$, and $\delta = 0.79$. In (b), $p_1 = 0.06$, $q_1 = p_2 = 0$, $q_2 = 0.33$, $q_c = 0.04$, $\theta_1 = 0.63$, $r = 0.005$, and $\delta = 1$. In (c), $p_1 = 0.02$, $q_1 = 0.27$, $p_2 = 0$, $q_2 = 0.19$, $q_c = 0.004$, $\theta_1 = 0.05$, $r = 0.005$, and $\delta = 0.96$. In (d), $p_1 = 0.02$, $q_1 = 0.16$, $p_2 = q_2 = 0$, $q_c = 0.01$, $\theta_1 = 0.67$, $r = 0.005$, and $\delta = 0.46$.

by a recycle" for Adam Ant, (3) "a slowly developing 'sleeper' pattern" for Everclear, and (4) "a bell shape" for DINK. Figure 7 shows each segment's CV for the four CDs over time. In each case, we determine the social apportioning parameter δ by applying the endogenous method described in §2.3. All music CD data in Van den Bulte and Joshi (2007) are weekly, and thus the discount rate r is set to 0.5% accordingly.

Figure 7(a) shows the CV of adopters who purchase Blind Melon over time. Whereas the innovation and imitation parameters of the influentials segment are significant, those of the imitators segment are zero. As expected, the influentials segment has a higher CV than the imitators segment, mostly because the latter exerts no influence on other potential adopters and thus has zero IV. In fact, CV of the early influentials can be 20 times as large as the CV of the early imitators. Note that CV of the influentials segment drops rapidly over time. For example, an influential who buys Blind Melon immediately after launch has a CV that is more than twice as much as that of an influential who delays her purchase to the fifth week. This result suggests the importance of early influentials in the diffusion process.

Figure 7(b) presents the CV of Adam Ant adopters over time. The nonzero parameters for this diffusion process are the influentials segment's innovation parameter (p_1), the imitators segment's within-segment imitation parameter (q_2), and the cross-segment imitation parameter (q_c). Note that early influentials have a lower CV than early imitators, but this relationship reverses after about the 10th week. Because the imitators segment is affected by both cross-segment influence ($q_c = 0.04$) and within-segment influence ($q_2 = 0.33$), it experiences a much faster diffusion process than the influentials segment ($q_1 = 0$). The high within-segment imitation parameter of the imitators segment causes its early adopters to have a high IV. As a consequence, early adopters in the imitators segment have a much higher CV than those in the influentials segment. This relationship reverses after a certain time as the pool of potential adopters in the imitators segment rapidly shrinks as more potential adopters purchase the new product.

Figure 7(c) shows the CV of Everclear adopters over time. Because the cross-segment influence is small ($q_c = 0.00$), the two segments follow their own Bass-type diffusion processes. Note that the imitators segment's adoption is entirely driven by social contagion because its innovation parameter (p_2) is zero. Moreover, because δ is close to one, most of the CV derives from IV. As a consequence, the CV of the imitators segment, which has higher weight on the imitation parameter, far exceeds that of the influentials segment.

Figure 7(d) shows the CV of DINK adopters over time. Here, all purchases in the imitators segment are driven by the cross-segment social influence from the influentials segment (because $p_2 = q_2 = 0$). As a consequence, the influentials segment has a higher CV than the imitators segment at all times. Note that the CV of the influentials segment decreases much more rapidly than that of the imitators segment.

3. Purchase Acceleration

In this section, we study how a firm can actively control the social contagion process in order to maximize its total customer value and how the social apportioning parameter δ is affected as a consequence. Specifically, we examine the effect of offering introductory price discounts at product launch to a subset of customers in order to speed up the diffusion process. We term this introductory discount strategy *purchase acceleration*. We assume that customers within a target segment are exchangeable, so that the firm can randomly sample a group of potential adopters and offer them the introductory discount. As a consequence, purchase acceleration will not change the average propensity of innovation and imitation of the target segment. In addition, we assume that the introductory price discount is deep enough so that all targeted potential adopters are willing to purchase the new product immediately upon receiving the offer.

To make the model tractable, we first investigate the case where the imitators segment receives social influence only from the influentials segment and thus has no within-segment social influence behavior; i.e., $q_2 = 0$. We shall then conduct an extensive numerical simulation to investigate purchase acceleration in the general case where $q_2 > 0$.

Case 1. *No within-segment imitation for the imitators segment* ($q_2 = 0$). If the imitators segment receives only cross-segment social influence, it is optimal for the firm to offer the introductory price discount only to the influentials segment. We are interested in determining the optimal number of influentials to receive the price discount; i.e., $F_1(0)$ may be greater than 0. We use l to denote the unit-selling price, c the unit production cost, and d the unit price discount offered to the selected group of influentials whom we shall call *invited customers*.

When $q_2 = 0$ and no customers are invited, the cumulative adoption functions become

$$F_1(t \mid 0) = \frac{1 - e^{-(p_1+q_1)t}}{1 + (q_1/p_1)e^{-(p_1+q_1)t}},$$

$$F_2(t \mid 0) = 1 - e^{-(p_2+q_c)t + (q_c/q_1)\ln(1+(q_1/p_1)F_1(t\mid 0))}.$$

Let N denote the size of the combined population. Then, $N_1 = \theta_1 N$ and $N_2 = \theta_2 N$ are the total number of influentials and the total number of imitators, respectively. We shall target M_1 potential adopters in the influentials segment. If no introductory price discount is offered, the total PV of type i is

$$(l-c)N_i\left(\int_0^\infty \mathrm{PV}_i(t\mid 0)f_i(t\mid 0)\,dt\right),$$

where $\mathrm{PV}_i(t\mid 0)$ follows from Equations (11) and (13) when no influentials are offered introductory price discounts at product launch (i.e., $F_1(0)=0$).

Similarly, the total IV of type i is given by

$$(l-c)N_i\left(\int_0^\infty \mathrm{IV}_i(t\mid 0)f_i(t\mid 0)\,dt\right).$$

Let us assume that M_1 influentials are selected and offered an introductory price discount d at product launch. Their immediate adoption of the new product starts the new product diffusion process with a pool of M_1 adopters. With $F_1(0)=M_1/N_1$, the cumulative adoption functions of the two segments become

$$F_1(t\mid M_1)=\frac{1-Ae^{-(p_1+q_1)t}}{1+(q_1/p_1)Ae^{-(p_1+q_1)t}}, \tag{18}$$

$$F_2(t\mid M_1)=1-e^{-(p_2+q_c)t+(q_c/q_1)\ln((p_1+q_1A)/(p_1+q_1Ae^{-(p_1+q_1)t}))}, \tag{19}$$

where $A=(p_1(N_1-M_1))/(p_1N_1+q_1M_1)$. The corresponding instantaneous adoption rate of each segment is

$$f_1(t\mid M_1)=\frac{(N_1-M_1)(p_1N_1+q_1M_1)(p_1+q_1)^2e^{-(p_1+q_1)t}}{(p_1N_1+q_1M_1+q_1(N_1-M_1)e^{-(p_1+q_1)t})^2}, \tag{20}$$

$$f_2(t\mid M_1)=e^{-(p_2+q_c)t+(q_c/q_1)\ln((p_1+q_1A)/(p_1+q_1Ae^{-(p_1+q_1)t}))}\cdot\left(p_2+q_c-\frac{q_c(p_1+q_1)Ae^{-(p_1+q_1)t}}{p_1+q_1Ae^{-(p_1+q_1)t}}\right). \tag{21}$$

With a unit price discount d, the firm sells to each invited customer at a price $l-d$ and thus collects a margin of $l-d-c$ from each sale. Note that $l-d-c$ may not be positive (e.g., each invited customer is given a free product; $l-d=0$). A negative value of $l-d-c$ indicates that the firm has to incur a loss for each invited customer. As the discount is offered at product launch, the PV of each invited customer is the profit (or loss) she brings in (i.e., $l-d-c$). Hence, the total PV of the targeted M_1 influentials is

$$(l-d-c)M_1. \tag{22}$$

The M_1 adopters start the social influence process at product launch. Thus, their total IV is

$$(l-c)M_1IV_1(0\mid M_1), \tag{23}$$

where

$$IV_1(t\mid M_1)=\delta\left(q_1\int_t^\infty e^{-rs}(1-F_1(s\mid M_1))\,ds+q_c\bar{\theta}\int_t^\infty e^{-rs}(1-F_2(s\mid M_1))\,ds\right).$$

The above equation follows directly from Equation (12) by substituting $F_i(t)$ with $F_i(t\mid M_1)$ $(i=1,2)$ from Equations (18) and (19).

The total PV of the noninvited influentials is

$$(l-c)N_1\int_0^\infty \mathrm{PV}_1(t\mid M_1)f_1(t\mid M_1)\,dt, \tag{24}$$

and the total PV of the imitators segment is

$$(l-c)N_2\int_0^\infty \mathrm{PV}_2(t\mid M_1)f_2(t\mid M_1)\,dt, \tag{25}$$

where

$$\mathrm{PV}_1(t\mid M_1)=e^{-rt}\left(1-\frac{\delta q_1F_1(t\mid M_1)}{p_1+q_1F_1(t\mid M_1)}\right),$$

$$\mathrm{PV}_2(t\mid M_1)=e^{-rt}\left(1-\frac{\delta q_cF_1(t\mid M_1)}{p_2+q_cF_1(t\mid M_1)}\right).$$

Note that the above two equations are derived from Equations (11) and (13) by replacing $F_i(t)$ with $F_i(t\mid M_1)$ $(i=1,2)$. Similarly, the total IV of the noninvited influentials is

$$(l-c)N_1\int_0^\infty \mathrm{IV}_1(t\mid M_1)f_1(t\mid M_1)\,dt. \tag{26}$$

Finally, the IV of the imitators segment is always zero because imitators do not exert social influence on others. Table 4 provides a summary of the PV, IV, and CV by customer type with or without purchase acceleration via the introductory discount. We are interested in determining the optimal number of influentials to target and invite. Our optimization problem is formally stated as follows:

$$\text{(P)}\quad \pi_0^*=\max_{0\le M_1\le N_1}\;(l-c)N\int_0^\infty e^{-rt}(\theta_1f_1(t\mid M_1)+\theta_2f_2(t\mid M_1))\,dt+(l-d-c)M_1, \tag{27}$$

where $f_1(t\mid M_1)$ and $f_2(t\mid M_1)$ are Equations (20) and (21), respectively. The objective of the optimization problem is the total profit, or total customer value, from the entire customer base (the summation of Equations (22)–(26)). The firm wishes to maximize its total profit subject to the diffusion dynamics described in Equations (18) and (19) with an initial pool of M_1 influentials as invited customers at product launch. Proposition 6 characterizes the solution to the optimization problem (P).

Table 4 Customer PV, IV, and CV With and Without Purchase Acceleration When $q_2 = 0$

	No purchase acceleration	Purchase acceleration
Type 1		
PV	$(l-c)N_1 \int_0^\infty \mathrm{PV}_1(t \mid 0) f_1(t \mid 0)\,dt$	$(l-c)N_1 \int_0^\infty \mathrm{PV}_1(t \mid M_1) f_1(t \mid M_1)\,dt + (l-d-c)M_1$
IV	$(l-c)N_1 \int_0^\infty \mathrm{IV}_1(t \mid 0) f_1(t \mid 0)\,dt$	$(l-c)(N_1 \int_0^\infty \mathrm{IV}_1(t \mid M_1) f_1(t \mid M_1)\,dt + IV_1(0 \mid M_1)M_1)$
Type 2		
PV	$(l-c)N_2 \int_0^\infty \mathrm{PV}_2(t \mid 0) f_2(t \mid 0)\,dt$	$(l-c)N_2 \int_0^\infty \mathrm{PV}_2(t \mid M_1) f_2(t \mid M_1)\,dt$
IV	0	0
Total		
PV	$(l-c)N \sum_i \theta_i \int_0^\infty \mathrm{PV}_i(t \mid 0) f_i(t \mid 0)\,dt$	$(l-c)N \sum_i \theta_i \int_0^\infty \mathrm{PV}_i(t \mid M_1) f_i(t \mid M_1)\,dt + (l-d-c)M_1$
IV	$(l-c)N\theta_1 \int_0^\infty \mathrm{IV}_1(t \mid 0) f_1(t \mid 0)\,dt$	$(l-c)(N\theta_1 \int_0^\infty \mathrm{IV}_1(t \mid M_1) f_1(t \mid M_1)\,dt + IV_1(0 \mid M_1)M_1)$
CV	$(l-c)N \sum_i \theta_i \int_0^\infty e^{-rt} f_i(t \mid 0)\,dt$	$(l-c)N \sum_i \theta_i \int_0^\infty e^{-rt} f_i(t \mid M_1)\,dt + (l-d-c)M_1$

Proposition 6. *We define*

$$G(t, M_1) = e^{-rt} N\left(\theta_1 \frac{\partial F_1(t \mid M_1)}{\partial M_1} + \theta_2 \frac{\partial F_2(t \mid M_1)}{\partial M_1}\right),$$

where

$$\frac{\partial F_1(t \mid M_1)}{\partial M_1} = \frac{N_1(p_1+q_1)^2 e^{-(p_1+q_1)t}}{(p_1N_1 + q_1M_1 + q_1(N_1-M_1)e^{-(p_1+q_1)t})^2}, \tag{28}$$

$$\frac{\partial F_2(t \mid M_1)}{\partial M_1} = e^{-(p_2+q_c)t + (q_c/q_1)\ln((p_1+q_1A)/(p_1+q_1Ae^{-(p_1+q_1)t}))} \times \frac{q_c p_1^2 (p_1+q_1) N_1 (1-e^{-(p_1+q_1)t})}{(p_1+q_1A)(p_1+q_1Ae^{-(p_1+q_1)t})(p_1N_1+q_1M_1)^2}. \tag{29}$$

Then the optimal solution M_1^* for (P) is uniquely determined as follows:

1. $M_1^* = 0$ *if* $\int_0^\infty G(t, 0)\,dt \le d/((l-c)r)$.
2. $M_1^* = N_1$ *if* $\int_0^\infty G(t, N_1)\,dt \ge d/((l-c)r)$.
3. $0 < M_1^* < N_1$ *satisfies* $\int_0^\infty G(t, M_1^*) = d/((l-c)r)$ *otherwise*.

To illustrate Proposition 6, we use the average parametric values of the annual data from Van den Bulte and Joshi (2007) and artificially set q_2 to zero. Applying Proposition 3, we obtain $\delta = 0.739$. We conduct a numerical experiment to determine M_1^*. We find that it is optimal to invite 5.4% of the total potential adopters (i.e., $M_1^*/N = 0.054$). Moreover, under purchase acceleration, δ increases to 0.742, which suggests that more purchase value is credited to the influencers under purchase acceleration. Next, we evaluate the total PV and IV of each type of customer depending on whether purchase acceleration is implemented by offering price discounts to M_1^* potential adopters at product launch. Using the formulas presented in Table 4, we obtain the results as shown in Table 5.[4]

[4] In the numerical results in Table 5, N is normalized to 1 because the ratio of the optimal number of invited customers to the size of the combined population (M_1^*/N) stays constant regardless of the size of N. In addition, we assume that the firm sells the new product at its unit production cost (i.e., $l-d=c$).

Table 5 Customer PV, IV, and CV With and Without Purchase Acceleration

	No purchase acceleration	Purchase acceleration
Influentials		
PV	34.10	29.47
IV	82.52	90.78
Imitators		
PV	19.93	20.53
IV	0	0
Total		
PV	54.03	50.00
IV	82.52	90.78
CV	136.55	140.78

Note. $p_1 = 0.06$, $q_1 = 0.65$, $p_2 = 0.02$, $q_2 = 0$, $q_c = 0.62$, $\theta_1 = 0.54$, $r = 0.1$, $N = 1$, $M_1^* = 0.054$, $l-d = 200$, and $c = 200$.

Table 5 reveals several insights. First, the total PV of the influential segment decreases but its IV increases. The net effect is a slight increase in its CV. Second, the PV (and hence CV) of the imitators segment increases because they adopt the new product faster because of greater social influence by the influentials segment. Third, the firm increases its total customer value by 3.1% (from 136.55 to 140.78) through purchase acceleration.

Under purchase acceleration, the firm loses in the influentials segment's PV but gains from its increased IV as well as the imitators segment's increased PV. We illustrate this insight further by examining the value of a customer who adopts at the mean time of adoption. We use $\hat{T}$ to denote the time when the product is adopted. We define

$$\mathbb{E}\hat{T} = \int_{t=0}^{\infty} t f_m(t \mid M_1)\,dt,$$

where $f_m(t \mid M_1)$ derives from Equations (20) and (21). We compare the PV, IV, and CV at the mean time of adoption with and without purchase acceleration in Table 6. Our result shows a boost in the PV, IV, and CV of the customer who adopts at the mean adoption time if the firm implements purchase acceleration.

Table 6 Customer PV, IV, CV, and Firm Profit at Mean Time of Adoption

	No purchase acceleration	Purchase acceleration	Increase (%)
δ	0.739	0.742	—
M_1^*/N (%)	0	5.4	—
Profit	136.55	140.78	3.10
$\mathbb{E}\hat{T}$	4.04	2.98	—
$\mathrm{PV}_m(\mathbb{E}\hat{T})$	44.18	48.32	9.37
$\mathrm{IV}_m(\mathbb{E}\hat{T})$	46.00	49.63	7.89
$\mathrm{CV}_m(\mathbb{E}\hat{T})$	90.18	97.95	8.62

Note. $p_1 = 0.06$, $q_1 = 0.65$, $p_2 = 0.02$, $q_2 = 0$, $q_c = 0.62$, $\theta_1 = 0.54$, $r = 0.1$, $N = 1$, $l - d = 200$, and $c = 200$.

Case 2. *Positive within-segment imitation for the imitators segment* ($q_2 > 0$). We now consider the general case where the imitators segment has a positive within-segment imitation parameter (i.e., $q_2 > 0$). We use two new product diffusion scenarios to illustrate how the promotion dollars should be allocated to the influentials and the imitators segments. We first classify the 11 product categories of annual data with positive q_2 from Van den Bulte and Joshi (2007) into two clusters using the estimated parametric values and use the cluster mean to investigate each cluster's product diffusion scenario. Tables 7 and 8 show how the optimal number of invitees from both segments (M_1^* and M_2^*) and the total profit vary with the level of price discount offered in these two product diffusion scenarios.

In both Tables 7 and 8, we find that a more costly price discount (i.e., higher discount to margin ratio) reduces the number of invited potential adopters and the amount by which the total profit increases. This is to be expected because offering an introductory price discount has two competing effects: (1) an acceleration effect and (2) a cannibalization effect (Bawa and Shoemaker 2004). On one hand, offering introductory price discounts to more potential adopters will accelerate the spread of social contagion, and thus potential adopters are likely to make purchases earlier than they would otherwise have. On the other hand, this strategy also reduces the number of potential adopters who would have bought the new product at the regular price. The number of invitees is determined so that the cannibalization effect does not override the acceleration effect, hence yielding the maximum profit for the firm.

Table 7 shows that most of the invited potential adopters come from the influentials segment in new product diffusion scenario 1. Specifically, none of the imitators segment should be invited if the discount/margin ratio exceeds 90%. Because the cross-segment imitation parameter is almost zero, the two segments experience their own Bass-type diffusion processes. Note that the influentials segment in this scenario is the majority, and its within-segment imitation parameter is significant ($q_1 = 0.83$) whereas its innovation parameter is almost zero ($p_1 = 0.00$). Hence, it is optimal for the firm to spend more on influentials than on imitators. Our model confirms this intuition by suggesting that the firm should allocate most of the discount dollars to the influentials segment in order to maximize the total customer value.

Table 7 Product Diffusion Scenario 1

Discount/margin (%)	Type 1 invitees (%)	Type 2 invitees (%)	Profit increase (%)
60	13.3	0.3	52.5
70	11.2	0.2	49.9
80	9.6	0.1	47.8
90	8.5	0	45.9
100	7.5	0	44.3
110	6.8	0	42.8
120	6.1	0	41.5
130	5.6	0	40.3
150	4.8	0	38.2
170	4.1	0	36.4
200	3.5	0	34.0

Note. $p_1 = 0.00$, $q_1 = 0.83$, $p_2 = 0.10$, $q_2 = 2.63$, $q_c = 0.00$, $\theta_1 = 0.85$, $r = 0.1$, and $l - c = 200$.

Table 8 shows that potential adopters should be invited from both segments in new product diffusion scenario 2. Compared with new product diffusion scenario 1, the number of invitees from the imitators segment is significant. Indeed, the number of invited customers from the imitators segment is about half the number from the influentials segment. This is because the proportion of the imitators in the combined population is about the same as that of the influentials ($\theta_1 = 0.46$). Because the cross-segment imitation parameter is significant ($q_c = 0.21$), the influentials segment plays a more important role in spreading social contagion. Our model therefore reveals that it is optimal for the firm to allocate its limited promotion dollars more to the influentials

Table 8 Product Diffusion Scenario 2

Discount/margin (%)	Type 1 invitees (%)	Type 2 invitees (%)	Profit increase (%)
40	13.2	7.0	10.8
50	9.8	5.0	8.3
60	7.5	3.8	6.4
70	5.8	3.0	5.0
80	4.6	2.4	3.9
90	3.6	2.0	3.0
100	2.8	1.7	2.2
110	2.1	1.4	1.7
210	0	0	0

Note. $p_1 = 0.08$, $q_1 = 0.62$, $p_2 = 0.00$, $q_2 = 0.78$, $q_c = 0.21$, $\theta_1 = 0.46$, $r = 0.1$, and $l - c = 200$.

segment while not completely ignoring the imitators segment.

One might be tempted to conjecture that it is always optimal to allocate more discount dollars to the influentials segment because it is connected with more potential adopters. However, this is not the case. For instance, when we artificially set the proportion of the influentials to a low level (e.g., $\theta_1 = 0.1$) in new product diffusion scenario 1, we find that it is optimal to invite fewer potential adopters from the influentials segment than from the imitators segment. In sum, our model suggests that, depending on the diffusion parameters, the firm should invite potential adopters from either segment in a way such that the acceleration effect and the cannibalization effect are optimally balanced.

Purchase acceleration also influences the social apportioning parameter δ. To better explain the relationship, we examine two competing effects purchase acceleration has on δ. First, because the product life cycle is compressed under purchase acceleration, at any point in time, the remaining pool of potential adopters is smaller, and the discounted profit generated by a customer is shared by more influencers who have previously adopted the product. We call this effect the decreased share effect, which works toward decreasing δ. On the other hand, from an influencer's perspective, each influencee on average adopts earlier and generates a higher discounted profit for the influencer to share. We call this effect the increased value effect, which works toward increasing δ.

To see how δ responds to the interplay between the decreased share and increased value effects, we compare δ before and after purchase acceleration in each of the 32 products in Van den Bulte and Joshi (2007). Among the 20 products where it is optimal to invite nonzero customers from either segment, δ increases in 13 products and decreases in 7 products. Interestingly, all seven products in which δ decreases as a result of purchase acceleration have a very small influentials segment's innovation parameter ($p_1 \leq 0.001$). This result suggests that when the diffusion process of the influentials segment is primarily driven by social contagion, the decreased share effect is stronger than the increased value effect and as a result δ decreases. When p_1 is large, the increased value effect overrides the decreased share effect, and hence δ increases under purchase acceleration.

4. Conclusion

In this paper, we develop a model framework for quantifying the value of a customer in the diffusion of new product where social contagion is prevalent. Stated formally, we posit that CV = PV + IV. Our model is able to decompose a customer value into her purchase and influence values by her type (whether the influentials or imitators segment) and by her time of adoption. We believe this explicit account of a customer's influence value is crucial to developing a deeper understanding of new product adoption, especially in the Web 2.0 world, where social contagion is widespread and managing it is a firm's key priority.

Our model allows for customer heterogeneity by building on the two-segment influential–imitator asymmetric influence model by Van den Bulte and Joshi (2007). We derive closed-form expressions for the PV, IV, and CV for each segment. In addition, we develop a new endogenous method of apportioning customer value so that proper credits can be given to influencers over the course of the product life cycle. We investigate how the PV, IV, and CV vary with the diffusion parameters and demonstrate the dynamics of CV for each segment under various new product diffusion scenarios reported in Van den Bulte and Joshi (2007).

Our model can be used to determine the optimal number of potential adopters to offer introductory price discounts in order to maximize firms' total customer value. We show that firms can often increase their total customer value by trading lower PV with high IV for the total customer base. The purchase acceleration strategy also has an interesting effect on the social apportioning parameter δ. It tends to decrease δ when the influentials segment has a very small innovation parameter and increase δ when the reverse is true.

Our CV = PV + IV framework yields two useful insights on managing new product diffusion. First, firms should pay more attention and allocate more resources to early adopters, because their CVs are significantly higher than the discounted profits they generate. Second, our model reveals that offering introductory discounts to a selected group of potential customers is a direct and powerful way to accelerate product purchases and boost firms' total customer value.

Our model paves the way for several future research directions. First, our model can be extended to study the impact of other marketing mix variables (such as price and product) on the dynamics of the PV, IV, and CV in new product diffusion. Second, it will be worthwhile to investigate these dynamics in a competitive setting when our model is extended to capture active rivalry (e.g., Savin and Terwiesch 2005). Third, it will be interesting to examine how customer purchase and influence values vary over time in a repeated purchase setting where existing customers may leave and switch to other firms. Finally, our modeling framework can be generalized to incorporate settings where the number of potential adopters can either grow or decline over time.

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Appendix
Generalized solutions to $F_1(t)$ and $F_2(t)$ are given by

$$F_1(t \mid M_1, M_2) = \frac{1 - Ae^{-(p_1+q_1)t}}{1+(q_1/p_1)Ae^{-(p_1+q_1)t}},$$

$$F_2(t \mid M_1, M_2) = 1 + \left(e^{-(p_2+q_2+q_c)t}\left(1+\frac{q_1}{p_1}Ae^{-(p_1+q_1)t}\right)^{-q_c/q_1}\right) \cdot \left[q_2\int_0^t e^{-(p_2+q_2+q_c)s}\left(1+\frac{q_1}{p_1}Ae^{-(p_1+q_1)s}\right)^{-q_c/q_1} ds - \frac{(1+(q_1/p_1)A)^{-q_c/q_1}}{1-M_2/N_2}\right]^{-1},$$

where $F_1(0 \mid M_1, M_2) = M_1/N_1$, $F_2(0 \mid M_1, M_2) = M_2/N_2$, and $A = p_1(N_1 - M_1)/(p_1N_1 + q_1M_1)$.

Solution for $F_1(t \mid M_1, M_2)$. To simplify notation, we hereafter omit the time argument from functions and write F_i instead of $F_i(t \mid M_1, M_2)$ $(i = 1, 2)$. $f_1 = (p_1 + q_1F_1)(1 - F_1)$ can be written as $dF_1/dt = (p_1 + q_1F_1)(1 - F_1)$. The solution to this differential equation is

$$F_1 = 1 + \frac{e^{-(p_1+q_1)t}}{-(q_1/(p_1+q_1))e^{-(p_1+q_1)t} + C}.$$

Since $F_1(0) = M1/N1$, we derive $C = q_1/(p_1 + q_1) - 1/(1 - M_1/N_1)$. Substituting C, we have

$$F_1 = \frac{1 - Ae^{-(p_1+q_1)t}}{1+(q_1/p_1)Ae^{-(p_1+q_1)t}},$$

where $A = p_1(N_1 - M_1)/(p_1N_1 + q_1M_1)$. Note that $A = 1$ when $M_1 = 0$.

Solution for $F_2(t \mid M_1, M_2)$. We have

$$\begin{aligned}\frac{dF_2}{dt} &= (p_2 + q_cF_1 + q_2F_2)(1 - F_2)\\ &= \left(p_2 + q_c\frac{1 - Ae^{-(p_1+q_1)t}}{1+(q_1/p_1)Ae^{-(p_1+q_1)t}}\right)\\ &\quad + \left(q_2 - p_2 - q_c\frac{1 - Ae^{-(p_1+q_1)t}}{1+(q_1/p_1)Ae^{-(p_1+q_1)t}}\right)F_2 - q_2F_2^2. \qquad (30)\end{aligned}$$

Equation (30) is a Ricatti equation of the general form $dF_2/dt = P(t) + Q(t)F_2 + R(t)F_2^2$, with

$$P(t) = p_2 + q_c\frac{1 - Ae^{-(p_1+q_1)t}}{1+(q_1/p_1)Ae^{-(p_1+q_1)t}},$$

$$Q(t) = q_2 - p_2 - q_c\frac{1 - Ae^{-(p_1+q_1)t}}{1+(q_1/p_1)Ae^{-(p_1+q_1)t}},$$

$$R(t) = -q_2.$$

We observe that $F_2 = 1$ is a potential solution. Let $z = 1/(F_2 - 1)$. Then $F_2 = 1 + 1/z$ and $dF_2/dt = -(1/z^2)(dz/dt)$. Equation (30) now becomes

$$-\frac{1}{z^2}\frac{dz}{dt} = P(t) + Q(t)\frac{z+1}{z} - q_2\frac{(z+1)^2}{z^2},$$

$$\frac{dz}{dt} = (-P(t) - Q(t) + q_2^2)z^2 - Q(t)z + 2q_2z + q_2, \qquad (31)$$

$$\frac{dz}{dt} = \left(q_2 + p_2 + q_c\frac{1 - Ae^{-(p_1+q_1)t}}{1+(q_1/p_1)Ae^{-(p_1+q_1)t}}\right)z + q_2.$$

Equation (31) is of the form $dz/dt + P_1(t)z = Q_1(t)$, with

$$P_1(t) = -\left(q_2 + p_2 + q_c\frac{1 - Ae^{-(p_1+q_1)t}}{1+(q_1/p_1)Ae^{-(p_1+q_1)t}}\right),$$

$$Q_1(t) = q_2.$$

Since F_2 is continuous in $[0, 1]$, z is continuous in $(-\infty, -1]$. Thus the general solution for Equation (31) is

$$z(t) = \frac{\int_{-\infty}^t R(s)Q_1(s)\,ds + C}{R(t)}, \qquad (32)$$

where $R(t) = e^{\int P_1(t)\,dt}$. Since

$$\int P_1(t)\,dt = -(p_2 + q_2 + q_c)t - \frac{q_c}{q_1}\ln\left(1+\frac{q_1}{p_1}Ae^{-(p_1+q_1)t}\right),$$

we get

$$R(t) = e^{-(p_2+q_2+q_c)t}\left(1+\frac{q_1}{p_1}Ae^{-(p_1+q_1)t}\right)^{-q_c/q_1}.$$

Hence,

$$\int_{-\infty}^t R(s)Q_1(s)\,ds = q_2\int_{-\infty}^t e^{-(p_2+q_2+q_c)s}\left(1+\frac{q_1}{p_1}Ae^{-(p_1+q_1)s}\right)^{-q_c/q_1} ds.$$

Substituting back into Equation (32), we obtain

$$z(t) = \frac{q_2\int_{-\infty}^t e^{-(p_2+q_2+q_c)s}\left(1+(q_1/p_1)Ae^{-(p_1+q_1)s}\right)^{-q_c/q_1} ds + C}{e^{-(p_2+q_2+q_c)t - (q_c/q_1)\ln(1+(q_1/p_1)Ae^{-(p_1+q_1)t})}}.$$

Transforming z back to F_2, we get

$$F_2 = 1 + \frac{e^{-(p_2+q_2+q_c)t}(1+(q_1/p_1)Ae^{-(p_1+q_1)t})^{-q_c/q_1}}{q_2\int_{-\infty}^t e^{-(p_2+q_2+q_c)s}(1+(q_1/p_1)Ae^{-(p_1+q_1)s})^{-q_c/q_1}\,ds + C}.$$

As $F_2(0 \mid M_1, M_2) = M_2/N_2$, we have

$$\frac{M_2}{N_2} = 1 + \frac{e^{-(q_c/q_1)\ln(1+(q_1/p_1)A)}}{q_2\int_{-\infty}^0 e^{-(p_2+q_2+q_c)s}(1+(q_1/p_1)Ae^{-(p_1+q_1)s})^{-q_c/q_1}\,ds + C}.$$

Then,

$$C = -\frac{e^{-(q_c/q_1)\ln(1+(q_1/p_1)A)}}{1-M_2/N_2} - q_2\int_{-\infty}^0 e^{-(p_2+q_2+q_c)s}\left(1+\frac{q_1}{p_1}Ae^{-(p_1+q_1)s}\right)^{-q_c/q_1} ds.$$

Thus,

$$F_2 = 1 + \left(e^{-(p_2+q_2+q_c)t}\left(1+\frac{q_1}{p_1}Ae^{-(p_1+q_1)t}\right)^{-q_c/q_1}\right) \cdot \left[q_2\int_0^t e^{-(p_2+q_2+q_c)s}\left(1+\frac{q_1}{p_1}Ae^{-(p_1+q_1)s}\right)^{-q_c/q_1} ds - \frac{(1+(q_1/p_1)A)^{-q_c/q_1}}{1-M_2/N_2}\right]^{-1},$$

where $A = p_1(N_1 - M_1)/(p_1N_1 + q_1M_1)$.

Proof of Equation (15).

$$\begin{aligned}\int_{t=0}^{\infty} \mathrm{PV}_1(t) f_1(t)\,dt &= \int_{t=0}^{\infty} e^{-rt}\left(1-\frac{\delta q_1 F_1(t)}{p_1+q_1F_1(t)}\right)(p_1+q_1F_1(t))(1-F_1(t))\,dt \\ &= \int_{t=0}^{\infty} e^{-rt}(p_1+(1-\delta)q_1F_1(t))(1-F_1(t))\,dt, \qquad (33)\end{aligned}$$

$$\begin{aligned}\int_{t=0}^{\infty} \mathrm{IV}_1(t) f_1(t)\,dt &= \delta q_1 \int_{t=0}^{\infty}\int_{s=t}^{\infty} e^{-rs}(1-F_1(s)) f_1(t)\,ds\,dt \\ &\quad + \delta q_c\bar{\theta}\int_{t=0}^{\infty}\int_{s=t}^{\infty} e^{-rs}(1-F_2(s)) f_1(t)\,ds\,dt \\ &= \delta q_1 \int_{s=0}^{\infty}\int_{t=0}^{s} f_1(t)\,dt e^{-rs}(1-F_1(s))\,ds \\ &\quad + \delta q_c\bar{\theta}\int_{s=0}^{\infty}\int_{t=0}^{s} f_1(t)\,dt e^{-rs}(1-F_2(s))\,ds \\ &= \int_{t=0}^{\infty} e^{-rt}(1-F_1(t))(\delta q_1 F_1(t))\,dt \\ &\quad + \int_{t=0}^{\infty} e^{-rt}(1-F_2(t))(\delta q_c\bar{\theta}F_1(t))\,dt, \qquad (34)\end{aligned}$$

$$\begin{aligned}\int_{t=0}^{\infty} \mathrm{PV}_2(t) f_2(t)\,dt &= \int_{t=0}^{\infty} e^{-rt}\left(1-\frac{\delta(q_cF_1(t)+q_2F_2(t))}{p_2+q_cF_1(t)+q_2F_2(t)}\right) \\ &\quad \cdot (p_2+q_cF_1(t)+q_2F_2(t))(1-F_2(t))\,dt \\ &= \int_{t=0}^{\infty} e^{-rt}(p_2+(1-\delta)(q_cF_1(t)+q_2F_2(t)))(1-F_2(t))\,dt, \qquad (35)\end{aligned}$$

$$\begin{aligned}\int_{t=0}^{\infty} \mathrm{IV}_2(t) f_2(t)\,dt &= \delta q_2 \int_{t=0}^{\infty}\int_{s=t}^{\infty} e^{-rs}(1-F_2(s)) f_2(t)\,ds\,dt \\ &= \delta q_2 \int_{s=0}^{\infty}\int_{t=0}^{s} f_2(t)\,dt e^{-rs}(1-F_2(s))\,ds \\ &= \int_{t=0}^{\infty} e^{-rt}(1-F_2(s))(\delta q_2 F_2(t))\,dt. \qquad (36)\end{aligned}$$

We multiply Equations (33) and (34) with θ_1 and multiply Equations (35) and (36) with θ_2; then we add these equations side by side. Equation (15) follows. □

Proof of Proposition 2. It suffices to show that $\mathrm{PV}_1(t)$, $\mathrm{PV}_2(t)$, $\mathrm{IV}_1(t)$, and $\mathrm{IV}_2(t)$ are all decreasing and convex in t. The claim for $\mathrm{CV}_1(t)$ and $\mathrm{CV}_2(t)$ follows since $\mathrm{CV}_i(t) = \mathrm{PV}_i(t) + \mathrm{IV}_i(t)$ $(i=1,2)$.

To prove that $\mathrm{PV}_1(t)$ is decreasing and convex in t, we need to show that $\mathrm{PV}_1'(t) < 0$ and $\mathrm{PV}_1''(t) > 0$. We have

$$\mathrm{PV}_1'(t) = -e^{-rt}\left(r\left(1-\frac{\delta q_1F_1(t)}{p_1+q_1F_1(t)}\right)+\frac{\delta p_1 q_1(1-F_1(t))}{p_1+q_1F_1(t)}\right) < 0,$$

because both terms inside the bracket are positive. Moreover,

$$\mathrm{PV}_1''(t) = r(-\mathrm{PV}_1'(t)) + e^{-rt}\frac{\delta p_1 q_1 f_1(t)(r+p_1+q_1\theta)}{(p_1+q_1F_1(t))^2} > 0.$$

Hence, $\mathrm{PV}_1(t)$ is decreasing and convex in t. Similarly for $\mathrm{PV}_2(t)$, we have

$$\begin{aligned}\mathrm{PV}_2'(t) &= -e^{-rt}\left(r\left(1-\frac{\delta(q_cF_1(t)+q_2F_2(t))}{p_2+q_cF_1(t)+q_2F_2(t)}\right)\right. \\ &\quad \left. + \frac{\delta p_2(q_cf_1(t)+q_2f_2(t))}{(p_2+q_cF_1(t)+q_2F_2(t))^2}\right) < 0,\end{aligned}$$

and $\mathrm{PV}_2''(t) > 0$, proving that $\mathrm{PV}_2(t)$ is decreasing and convex in t.

Next, $\mathrm{IV}_1(t)$ is decreasing and convex in t because

$$\mathrm{IV}_1'(t) = -e^{-rt}\delta q_1(1-F_1(t)) - e^{-rt}\delta q_c\bar{\theta}(1-F_2(t)) < 0,$$

and

$$\begin{aligned}\mathrm{IV}_1''(t) &= re^{-rt}\delta q_1(1-F_1(t)) + e^{-rt}\delta q_1 f_1(t) \\ &\quad + re^{-rt}\delta q_c\bar{\theta}(1-F_2(t)) + e^{-rt}\delta q_c\bar{\theta}f_2(t) > 0.\end{aligned}$$

Finally, $\mathrm{IV}_2(t)$ is decreasing and convex in t because

$$\mathrm{IV}_2'(t) = -e^{-rt}\delta q_2(1-F_2(t)) < 0,$$

and

$$\mathrm{IV}_2''(t) = re^{-rt}\delta q_2(1-F_2(t)) + e^{-rt}\delta q_2 f_2(t) > 0. \quad \square$$

Proof of Proposition 3. Note that

$$\mathrm{CV}_m(\mathbf{p},\mathbf{q}) = \sum_{i=1}^{2}\theta_i\int_{t=0}^{\infty}\mathrm{PV}_i(t)f_i(t)\,dt + \sum_{i=1}^{2}\theta_i\int_{t=0}^{\infty}\mathrm{IV}_i(t)f_i(t)\,dt. \qquad (37)$$

From Equation (15), we know that

$$\mathrm{CV}_m(\mathbf{p},\mathbf{0}) = \sum_{i=1}^{2}\theta_i\int_{t=0}^{\infty}e^{-rt}f_i(t\mid\mathbf{p},\mathbf{0})\,dt, \qquad (38)$$

where $f_i(t\mid\mathbf{p},\mathbf{0})$ $(i=1,2)$ is the instantaneous adoption function for type i in the hypothetical population obtained by suppressing social influence. From Equations (3) and (4), $f_i(t\mid\mathbf{p},\mathbf{0})$ $(i=1,2)$ are determined as

$$f_1(t\mid\mathbf{p},\mathbf{0}) = p_1e^{-p_1t}, \qquad (39)$$

$$f_2(t\mid\mathbf{p},\mathbf{0}) = p_2e^{-p_2t}. \qquad (40)$$

Substituting Equations (37)–(40) into Equation (16), we have

$$\frac{\theta_1p_1}{p_1+r}+\frac{\theta_2p_2}{p_2+r} = \sum_{i=1}^{2}\theta_i\int_{t=0}^{\infty}\mathrm{PV}_i(t)f_i(t)\,dt. \qquad (41)$$

Substituting the right-hand side (RHS) of Equation (41) with Equations (11) and (13), we show that δ is determined by Equation (17).

Next we need to show that such δ uniquely exists within the range of $[0,1]$. Note that the left-hand side (LHS) of Equation (17) in Proposition 3 is independent of δ and the RHS is decreasing in δ. When $\delta=0$, the RHS becomes $\theta_1\int_{t=0}^{\infty}e^{-rt}f_1(t)\,dt + \theta_2\int_{t=0}^{\infty}e^{-rt}f_2(t)\,dt$, where $f_1(t)$ and $f_2(t)$ are, respectively, the instantaneous adoption rates of types 1 and 2 segments with social contagion. As purchase is accelerated in the presence of social influence, LHS $\le$ RHS.

When $\delta=1$, the RHS becomes $\theta_1\int_{t=0}^{\infty}e^{-rt}p_1(1-F_1(t))\,dt + \theta_2\int_{t=0}^{\infty}e^{-rt}p_2(1-F_2(t))\,dt$, where $F_1(t)$ and $F_2(t)$ are, respectively, the cumulative adoption functions of types 1 and 2 segments in the presence of social influence. The LHS is in fact equal to

$$\theta_1\int_{t=0}^{\infty}e^{-rt}p_1(1-\hat{F}_1(t))\,dt + \theta_2\int_{t=0}^{\infty}e^{-rt}p_2(1-\hat{F}_2(t))\,dt,$$

where $\hat{F}_1(t)$ and $\hat{F}_2(t)$ are, respectively, the cumulative adoption functions of types 1 and 2 segments in the hypothetical population. Because $F_1(t) \geq \hat{F}_1(t)$ and $F_2(t) \geq \hat{F}_2(t)$ for all t, we have LHS $\geq$ RHS when $\delta = 1$.

Therefore, given a set of innovation parameters ($\mathbf{p}$), imitation parameters ($\mathbf{q}$), and other parameters (θ_1, θ_2, and r), the social apportioning parameter δ can be uniquely and endogenously determined by Equation (17) within the desired range of $[0, 1]$. $\square$

Proof of Proposition 4. (1) To show that $\mathrm{PV}_1(t)$ increases with p_1, it suffices to show that $\partial \mathrm{PV}_1(t)/\partial p_1 > 0$ for all $t > 0$. We know that

$$\frac{\partial \mathrm{PV}_1(t)}{\partial p_1} = e^{-rt} \frac{q_1 \delta R(t)}{(p_1 + q_1)^2}, \tag{42}$$

where $R(t) = 1 - (p_1 t + q_1 t + 1) e^{-(p_1+q_1)t}$. Note that $R(0) = 0$. Moreover, $R'(t) = t(p_1 + q_1)^2 e^{-(p_1+q_1)t} > 0$ for $t > 0$. Therefore $R(t) > 0$ for all $t > 0$. It follows that $\partial \mathrm{PV}_1(t)/\partial p_1 > 0$ for all $t > 0$.

To show that $\mathrm{PV}_2(t)$ decreases with p_1, we first let $Y(t) = q_c F_1(t) + q_2 F_2(t)$. Then Equation (13) can be written as

$$\mathrm{PV}_2(t) = e^{-rt}\left(1 - \frac{\delta Y(t)}{p_2 + Y(t)}\right).$$

Its derivative with respect to p_1 is

$$\begin{aligned}\frac{\partial \mathrm{PV}_2(t)}{\partial p_1} &= -\delta e^{-rt} \frac{\partial}{\partial p_1}\left(\frac{Y(t)}{p_2 + Y(t)}\right) \\ &= -\delta e^{-rt} \frac{\partial}{\partial Y(t)}\left(\frac{Y(t)}{p_2 + Y(t)}\right) \cdot \frac{\partial Y(t)}{\partial p_1}.\end{aligned}$$

It is obvious that both $F_1(t)$ and $F_2(t)$ increase with p_1 from Equations (1) and (2). (Intuitively, the higher the influentials segment's innovation is, the faster the diffusion process becomes in either segment.) Hence, $\partial Y(t)/\partial p_1 > 0$. Also, $(\partial/\partial Y(t))(Y(t)/(p_2 + Y(t))) = p_2/(p_2 + Y(t))^2 > 0$. It follows that $\partial \mathrm{PV}_2(t)/\partial p_1 < 0$.

We can show that both $\mathrm{IV}_1(t)$ and $\mathrm{IV}_2(t)$ decrease with p_1 because

$$\begin{aligned}\frac{\partial \mathrm{IV}_1(t)}{\partial p_1} &= -\delta q_1 \int_t^\infty e^{-rs} \frac{\partial F_1(s)}{\partial p_1}\, ds \\ &\quad - \delta q_c \bar{\theta} \int_t^\infty e^{-rs} \frac{\partial F_2(s)}{\partial p_1}\, ds < 0,\end{aligned} \tag{43}$$

and

$$\frac{\partial \mathrm{IV}_2(t)}{\partial p_1} = -\delta q_2 \int_t^\infty e^{-rs} \frac{\partial F_2(s)}{\partial p_1}\, ds < 0.$$

It follows that $\mathrm{CV}_2(t)$ decreases with p_1 because $\mathrm{CV}_2(t) = \mathrm{PV}_2(t) + \mathrm{IV}_2(t)$.

To show how $\mathrm{CV}_1(t)$ varies with p_1, we let $H(t) = \partial \mathrm{CV}_1(t)/\partial p_1 = \partial \mathrm{PV}_1(t)/\partial p_1 + \partial \mathrm{IV}_1(t)/\partial p_1$, where $\partial \mathrm{PV}_1(t)/\partial p_1$ and $\partial \mathrm{IV}_1(t)/\partial p_1$ are from Equations (42) and (43). If $H(t) = 0$, we must have

$$\begin{aligned}&e^{-rt} \frac{1 - (p_1 t + q_1 t + 1) e^{-(p_1+q_1)t}}{(p_1 + q_1)^2} \\ &\quad = \int_t^\infty e^{-rs} \frac{\partial F_1(s)}{\partial p_1}\, ds + \frac{q_c \bar{\theta}}{q_1} \int_t^\infty e^{-rs} \frac{\partial F_2(s)}{\partial p_1}\, ds.\end{aligned} \tag{44}$$

We use $R_1(t)$ to denote the LHS and $R_2(t)$ to denote the RHS of Equation (44). Now we want to show that $R_1(t)$ is unimodal in t with a unique interior maximizer. Taking the first derivative with respect to t, we obtain

$$R_1'(t) = e^{-rt} q_1 \left(t e^{-(p_1+q_1)t} - r \frac{1 - (p_1 t + q_1 t + 1) e^{-(p_1+q_1)t}}{(p_1 + q_1)^2}\right).$$

Thus $R_1'(t) = 0$ if and only if $(p_1 + q_1)(p_1 + q_1 + r)t + r = re^{(p_1+q_1)t}$, the number of solutions of which is two; i.e., $t = 0$ and $t = t_s$ for some $t_s > 0$. Since $R_1'(t) > 0$ for $t \in (0, t_s)$ and $R_1'(t) < 0$ for $t \in (t_s, \infty)$, $R_1(t)$ is unimodal in t with a unique interior maximizer t_s. Hence, $R_1(t)$ increases with t for $t < t_s$ and decreases thereafter. Note, on the other hand, that $R_2(t)$ is strictly decreasing in t.

Since $R_1(0) = 0$ and $R_2(0) > 0$, $H(t) = R_1(t) - R_2(t)$ starts off at $t = 0$ from a negative value and eventually converges to zero because $\lim_{t\to\infty} R_1(t) = 0$ and $\lim_{t\to\infty} R_2(t) = 0$. Thus how $H(t)$ behaves over time depends on the number of intersections between $R_1(t)$ and $R_2(t)$. We define t_1 such that

$$\begin{aligned}t_1 = \min\Bigg\{t\colon\, &e^{-rt} \frac{1 - (p_1 t + q_1 t + 1) e^{-(p_1+q_1)t}}{(p_1 + q_1)^2} \\ &= \int_t^\infty e^{-rs} \frac{\partial F_1(s)}{\partial p_1}\, ds + \frac{q_c \bar{\theta}}{q_1} \int_t^\infty e^{-rs} \frac{\partial F_2(s)}{\partial p_1}\, ds\Bigg\}.\end{aligned}$$

Then for all $t < t_1$, $H(t)$ is negative and thus $\mathrm{CV}_1(t)$ decreases.

(2) First, it is clear that $\mathrm{PV}_1(t)$ is independent of p_2 because Equation (11) is not a function of p_2. Next, the first derivative of $\mathrm{PV}_2(t)$ with respect to p_2 is given by

$$\frac{\partial \mathrm{PV}_2(t)}{\partial p_2} = e^{-rt} \delta \frac{q_c F_1(t) + q_2 F_2(t) - q_2 p_2 (\partial F_2(t)/\partial p_2)}{(p_2 + q_c F_1(t) + q_2 F_2(t))^2}. \tag{45}$$

Thus $\mathrm{PV}_2(t)$ increases with p_2 if $F_2(t)/p_2 \geq \partial F_2(t)/\partial p_2$. This is true if $F_2(t)$ increases and is concave in p_2 for all t. Because we already showed that $F_2(t)$ is an increasing function of p_2, it suffices to show that $F_2(t)$ is concave in p_2 for all t. Let $F_2(t) = 1 + N(t)/D(t)$, where

$$N(t) = e^{-(p_2+q_2+q_c)t} \cdot (1 + (q_1/p_1) A e^{-(p_1+q_1)t})^{-q_c/q_1}$$

and

$$\begin{aligned}D(t) = q_2 \int_0^t &e^{-(p_2+q_2+q_c)s} (1 + (q_1/p_1) A e^{-(p_1+q_1)s})^{-q_c/q_1} \\ &\cdot ds - (1 + (q_1/p_1) A)^{-q_c/q_1} / (1 - M_2/N_2).\end{aligned}$$

Hence, $N(t)/D(t)$ is the second term on the RHS of Equation (4). Then,

$$\begin{aligned}\frac{\partial^2 F_2(t)}{\partial {p_2}^2} = \frac{1}{D(t)^2}\Bigg[&D(t) \frac{\partial^2 N(t)}{\partial {p_2}^2} - N(t) \frac{\partial^2 D(t)}{\partial {p_2}^2} - 2 \frac{\partial D(t)}{\partial p_2} \frac{\partial N(t)}{\partial p_2} \\ &+ 2 \frac{N(t)}{D(t)} \left(\frac{\partial D(t)}{\partial p_2}\right)^2\Bigg].\end{aligned}$$

It is easy to show that $N(t) > 0$, $D(t) < 0$, $\partial N(t)/\partial p_2 < 0$, $\partial D(t)/\partial p_2 < 0$, $\partial^2 N(t)/\partial {p_2}^2 > 0$, and $\partial^2 D(t)/\partial {p_2}^2 > 0$. Therefore, $\partial^2 F_2(t)/\partial {p_2}^2 < 0$ and $F_2(t)$ is concave in p_2 for all $t > 0$ as desired.

$F_2(t)$ increases with p_2 from Equation (2), and thereby $\mathrm{IV}_1(t)$ and $\mathrm{IV}_2(t)$ both decrease with p_2 because

$$\frac{\partial\,\mathrm{IV}_1(t)}{\partial p_2} = -\delta q_c\bar{\theta}\int_t^\infty e^{-rs}\frac{\partial F_2(s)}{\partial p_2}\,ds < 0,$$

and

$$\frac{\partial\,\mathrm{IV}_2(t)}{\partial p_2} = -\delta q_2\int_t^\infty e^{-rs}\frac{\partial F_2(s)}{\partial p_2}\,ds < 0. \tag{46}$$

It follows that $\mathrm{CV}_1(t)$ decreases with p_2.

From Equations (45) and (46), we have $\partial\mathrm{CV}_2(t)/\partial p_2 = 0$ if and only if

$$\frac{e^{-rt}((q_c/q_2)F_1(t)+F_2(t)-p_2(\partial F_2(t)/\partial p_2))}{(p_2+q_cF_1(t)+q_2F_2(t))^2} = \int_t^\infty e^{-rs}\frac{\partial F_2(s)}{\partial p_2}\,ds. \tag{47}$$

In Equation (47), LHS $=0$ at $t=0$ and $\lim_{t\to\infty}$ LHS $=0$. On the other hand, RHS >0 at $t=0$ and $\lim_{t\to\infty}$ RHS $=0$. Hence there exists a nonzero cutoff time such that $\mathrm{CV}_2(t)$ decreases with p_2 for all $t < t_2$. Such a cutoff time is defined as

$$t_2 = \min\left\{t\colon\ \frac{e^{-rt}((q_c/q_2)F_1(t)+F_2(t)-p_2(\partial F_2(t)/\partial p_2))}{(p_2+q_cF_1(t)+q_2F_2(t))^2} = \int_t^\infty e^{-rs}\frac{\partial F_2(s)}{\partial p_2}\,ds\right\}. \quad \square$$

Proof of Proposition 5. (1) To prove $\mathrm{PV}_1(t)$ decreases with q_1, it suffices to show that $\partial\,\mathrm{PV}_1(t)/\partial q_1 < 0$ for all $t>0$:

$$\frac{\partial\,\mathrm{PV}_1(t)}{\partial q_1} = e^{-rt}\frac{p_1\delta(R(t)-1)}{(p_1+q_1)^2},$$

where $R(t) = (1 - q_1t - q_1^2t/p_1)e^{-(p_1+q_1)t}$. Note that $\partial\,\mathrm{PV}_1(t)/\partial q_1 < 0$ if and only if $R(t)<1$. Moreover,

$$R'(t) = \frac{1}{p_1}(q_1t-1)(p_1+q_1)^2e^{-(p_1+q_1)t}.$$

Now observe that $R'(t)<0$ for $t<1/q_1$ and $R'(t)\geq 0$ thereafter. Thus $R(t)$ has a unique minimizer at $t=1/q_1$ and has its maximum at either $t=0$ or $t=\infty$. Note that $R(0)=1$ and $\lim_{t\to\infty}R(t)=0$. Hence, $R(t)<1$ for all $t>0$. It follows that $\partial\,\mathrm{PV}_1(t)/\partial q_1 < 0$ for all $t>0$.

To show that $\mathrm{PV}_2(t)$ decreases with q_1, we first let $Y(t) = q_cF_1(t)+q_2F_2(t)$. Then Equation (13) can be written as

$$\mathrm{PV}_2(t) = e^{-rt}\left(1-\frac{\delta Y(t)}{p_2+Y(t)}\right),$$

and thus

$$\frac{\partial\,\mathrm{PV}_2(t)}{\partial q_1} = -\delta e^{-rt}\frac{\partial}{\partial q_1}\left(\frac{Y(t)}{p_2+Y(t)}\right) = -\delta e^{-rt}\frac{\partial}{\partial Y(t)}\left(\frac{Y(t)}{p_2+Y(t)}\right)\cdot\frac{\partial Y(t)}{\partial q_1}.$$

It is clear from Equations (1) and (2) that $F_1(t)$ and $F_2(t)$ both increase with q_1. (Intuitively, the higher the influentials segment's within-segment imitation parameter is, the faster the diffusion process is in either segment.) Hence $\partial Y(t)/\partial q_1 > 0$. Also, $(\partial/\partial Y(t))(Y(t)/(p_2+Y(t))) = p_2/(p_2+Y(t))^2 > 0$. It follows that $\partial\,\mathrm{PV}_2(t)/\partial q_1 < 0$.

$\mathrm{IV}_2(t)$ decreases with q_1 because

$$\frac{\partial\mathrm{IV}_2(t)}{\partial q_1} = -\delta q_2\int_t^\infty e^{-rs}\frac{\partial F_2(s)}{\partial q_1}\,ds < 0.$$

It follows that $\mathrm{CV}_2(t)$ decreases with q_1 because $\mathrm{PV}_2(t)$ and $\mathrm{IV}_2(t)$ both decrease with q_1.

To see how $\mathrm{IV}_1(t)$ varies with q_1, we investigate

$$\frac{\partial\mathrm{IV}_1(t)}{\partial q_1} = \delta\int_{s=t}^\infty e^{-rs}\left(1-F_1(s)-q_1\frac{\partial F_1(s)}{\partial q_1}\right)ds + \delta q_c\bar{\theta}\int_{s=t}^\infty e^{-rs}\left(-\frac{\partial F_2(s)}{\partial q_1}\right)ds. \tag{48}$$

Let $H(s) = 1-F_1(s)-q_1(\partial F_1(s)/\partial q_1)$. Equating $H(s)$ to 0 yields

$$\frac{q_1^2}{p_1}e^{-(p_1+q_1)s} + 2q_1 + p_1 = p_1q_1s + q_1^2s. \tag{49}$$

Note that the LHS of Equation (49) is decreasing in s, whereas the RHS is increasing in s. Moreover, when $s=0$, the LHS is greater than the RHS (i.e., $H(0)>0$), and the RHS exceeds the LHS (i.e., $H(s)<0$) for large enough s. Thus $H(s)$ decreases from positive to negative and hits zero exactly once. Therefore, in Equation (48), the term inside the first integral is positive for small s and negative for large s. Thus the maximum value of the first integral is achieved at $t=0$. Thus, the left term on the RHS of Equation (48) decreases, then becomes negative for sure (even if it starts off as positive at $t=0$), and then converges to zero from negative as $t\to\infty$. Since we know $\partial F_2(s)/\partial q_1 > 0$ for all $s>0$, the right term on the RHS of Equation (48) starts off as negative, increases in t, and converges to zero from negative as $t\to\infty$ (note that it is never positive). Therefore, there must exist a cutoff time t_3 (which may be zero) such that $\mathrm{IV}_1(t)$ decreases with q_1 for all $t>t_3$. The cutoff time t_3 is defined as

$$t_3 = \max\left\{t\colon\ \int_t^\infty e^{-rs}\left(1-F_1(s)-q_1\frac{\partial F_1(s)}{\partial q_1} - q_c\bar{\theta}\frac{\partial F_2(s)}{\partial q_1}\right)ds = 0\right\}.$$

As a consequence, when $t>t_3$, $\mathrm{CV}_1(t)$ decreases with q_1 because $\mathrm{PV}_1(t)$ and $\mathrm{IV}_1(t)$ both decrease with it.

(2) $\mathrm{PV}_1(t)$ is independent of q_c because Equation (11) is not a function of q_c. To show that $\mathrm{PV}_2(t)$ decreases with q_c, we let $Y(t) = q_cF_1(t)+q_2F_2(t)$. Then Equation (13) can be written as

$$\mathrm{PV}_2(t) = e^{-rt}\left(1-\frac{\delta Y(t)}{p_2+Y(t)}\right),$$

and thus

$$\frac{\partial\,\mathrm{PV}_2(t)}{\partial q_c} = -\delta e^{-rt}\frac{\partial}{\partial q_c}\left(\frac{Y(t)}{p_2+Y(t)}\right) = -\delta e^{-rt}\frac{\partial}{\partial Y(t)}\left(\frac{Y(t)}{p_2+Y(t)}\right)\cdot\frac{\partial Y(t)}{\partial q_c}.$$

$F_2(t)$ increases with q_c because higher cross-segment imitation speeds up the diffusion process. Thus $\partial Y(t)/\partial q_c > 0$.

Moreover, $(\partial/\partial Y(t))(Y(t)/(p_2+Y(t))) = p_2/(p_2+Y(t))^2 > 0$. It follows that $\partial \mathrm{PV}_2(t)/\partial q_c < 0$.

$\mathrm{IV}_2(t)$ decreases with q_c because

$$\frac{\partial \mathrm{IV}_2(t)}{\partial q_c} = -\delta q_2 \int_t^\infty e^{-rs} \frac{\partial F_2(s)}{\partial q_c}\, ds < 0.$$

As a consequence, $\mathrm{CV}_2(t)$ decreases with q_c.

To see how $\mathrm{IV}_1(t)$ changes with q_c, we differentiate $\mathrm{IV}_1(t)$ with respect to q_c. We have

$$\frac{\partial \mathrm{IV}_1(t)}{\partial q_c} = \delta\bar{\theta} \int_{s=t}^\infty e^{-rs} \left(1 - F_2(s) - q_c \frac{\partial F_2(s)}{\partial q_c}\right) ds.$$

Thus $\mathrm{IV}_1(t)$ and $\mathrm{CV}_1(t)$ decrease with q_c if the above equation is negative.

(3) The proofs are similar to those in part (2). □

Proof of Proposition 6. We first rewrite the optimization problem (P) (see Equation (27)) as

$$\begin{aligned}
\pi_0^* = \max_{0\le M_1\le N_1} &(l-c)N\bigg(e^{-rt}\theta_1 F_1(t\mid M_1)\big|_{t=\infty} - e^{-rt}\theta_1 F_1(t\mid M_1)\big|_{t=0} \\
&+ r\int_0^\infty e^{-rt}\theta_1 F_1(t\mid M_1)\,dt\bigg) \\
&+ (l-c)N\bigg(e^{-rt}\theta_2 F_2(t\mid M_1)\big|_{t=\infty} - e^{-rt}\theta_2 F_2(t\mid M_1)\big|_{t=0} \\
&+ r\int_0^\infty e^{-rt}\theta_2 F_2(t\mid M_1)\,dt\bigg) + (l-d-c)M_1 \\
= \max_{0\le M_1\le N_1} &(l-c)N\bigg(r\int_0^\infty e^{-rt}(\theta_1 F_1(t) + \theta_2 F_2(t))\,dt - \frac{M_1}{N_1}\theta_1\bigg) \\
&+ (l-d-c)M_1,
\end{aligned}$$

where $F_1(t\mid M_1)$ and $F_2(t\mid M_1)$ are Equations (18) and (19), respectively. Taking the first-order condition with respect to M_1, we obtain

$$\int_0^\infty G(t, M_1)\,dt = \frac{d}{(l-c)r},$$

where

$$G(t, M_1) = e^{-rt} N\left(\frac{\theta_1 \partial F_1(t\mid M_1)}{\partial M_1} + \frac{\theta_2 \partial F_2(t\mid M_1)}{\partial M_1}\right).$$

It is easy to show that π_0^* is concave in $0 \le M_1 \le N_1$. Hence Proposition 6 naturally follows. □

References

Bass, F. M. 1969. A new product growth for model consumer durables. *Management Sci.* **15**(5) 215–227.

Bawa, K., R. Shoemaker. 2004. The effects of free sample promotions on incremental brand sales. *Marketing Sci.* **23**(3) 345–363.

Chatterjee, R., J. Eliashberg. 1990. The innovation diffusion process in a heterogeneous population: A micromodeling approach. *Management Sci.* **36**(9) 1057–1079.

Godes, D., D. Mayzlin. 2009. Firm-created word-of-mouth communication: Evidence from a field test. *Marketing Sci.* **28**(4) 721–739.

Goldenberg, J., O. Lowengart, D. Shapira. 2009. Zooming in: Self-emergence of movements in new product growth. *Marketing Sci.* **28**(2) 274–292.

Ho, T.-H., S. Savin, C. Terwiesch. 2002. Managing demand and sales dynamics in new product diffusion under supply constraint. *Management Sci.* **48**(2) 187–206.

Jain, D., V. Mahajan, E. Muller. 1995. An approach for determining optimal product sampling for the diffusion of a new product. *J. Product Innovation Management* **12**(2) 124–135.

Lehmann, D., M. Esteban-Bravo. 2006. When giving some away makes sense to jump-start the diffusion process. *Marketing Lett.* **17**(4) 243–254.

Libai, B., E. Muller, R. Peres. 2009. Sources of social value in word of mouth programs. Marketing Science Institute Working Paper 10-103, MSI, Cambridge, MA.

Marks, L. J., M. A. Kamins. 1988. The use of product sampling and advertising: Effects of sequence of exposure and degree of advertising claim exaggeration on consumers' belief strength, belief confidence and attitudes. *J. Marketing Res.* **25**(3) 266–281.

Muller, E., G. Yogev. 2006. When does the majority become a majority? Empirical analysis of the time at which main market adopters purchase the bulk of our sales. *Tech. Forecasting Soc. Change* **73**(9) 1107–1120.

Peres, R., E. Muller, V. Mahajan. 2010. Innovation diffusion and new product growth models: A critical review and research directions. *Internat. J. Res. Marketing* **27**(2) 91–106.

Savin, S., C. Terwiesch. 2005. Optimal product launch times in a duopoly: Balancing life-cycle revenues with product cost. *Oper. Res.* **53**(1) 26–47.

Van Ackere, A., D. J. Reyniers. 1995. Trade-ins and introductory offers in a monopoly. *RAND J. Econom.* **26**(1) 58–74.

Van den Bulte, C., Y. V. Joshi. 2007. New product diffusion with influentials and imitators. *Marketing Sci.* **26**(3) 400–421.