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Competitive location, production, and market selection

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Abstract

This paper investigates how firms should select their production sites, capacities and quantities under rivalry. There are assumed to be a finite number of discrete potential location sites and a finite number of discrete markets, which may or may not coincide. Firms first decide either simultaneously or sequentially whether and where to establish a production site. The fixed cost of opening a facility and the marginal cost of production both depend on where the facility is located. Firms then choose capacity and a production quantity for each market. Prices in each market are determined by the total quantity available at that location via the Cournot mechanism. This formulation thus addresses multi-market, oligopolistic spatial competition with heterogeneity in production and logistics costs.

We then analyze the Nash equilibria of the entry game and provide sufficient conditions for the existence of equilibria in the simultaneous entry game. At equilibrium, firms may not produce for all markets and may have limited market areas; however, these areas may overlap, so that there are multiple suppliers in any market. In general, there may not be a first mover advantage and early entrants may earn lower profits than later entrants.

Keywords: Location; Competition; Game theory

1. Introduction

1.1. The role of location

The role of location in competition is pervasive in the manufacturing sector. It is especially important in sectors where transportation and logistics costs play a large role. An obvious impact of these costs is to limit the market areas in which a plant can effectively compete, to some geographic region around it. This is, of course, the central issue addressed in the classic Hotelling (1929) model. For the manufacturing sector, in

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addition to location competition with respect to markets, it is also important to consider both the location dependent costs that arise due to raw material acquisition and the heterogeneous costs of operation at different sites. More generally, a location decision is one part of overall supply chain design, and location competition could be regarded as a core issue in supply chain competition.

Over the last few decades, even seemingly stable and mature manufacturing sectors have seen dramatic changes in the pattern of location and competition. A very well-known example is that of mini-mills in the steel industry, which have captured a substantial share of the market from the traditional integrated steel mills, in some portion of the market. Within the mini-mill environment too, changing technological capabilities, shifting markets and price movements affect location choices and competitive balance.

Another example is in the cement industry. In many developing economies, with economic growth, the pattern of market demand for cement in many countries has changed. Historically, the location of cement plants was often based on proximity to limestone deposits and coal. More recently, the need for proximity to markets has become more acute and new strategies for location are being devised. For example, when growth is driven by international trade, the demand for cement is often due to infrastructure development projects in ports and coastal regions.

Similar issues are surfacing in process industries such as chemicals, fertilizers, food processing, textile fibers, and aluminum. While location competition is also an important issue in most other manufacturing sectors, our models are especially appropriate for the kinds of process-oriented, commodity-manufacturing sectors mentioned above.

Location competition has been studied in different forms (see the comprehensive surveys by Friesz et al., 1989; Eiselt and Laporte, 1989; Eiselt et al., 1993). Among various competitive location models, network-based models may be classified by their complexity as captured by two criteria: (1) the number of entrants in the entry/location game, and (2) the consideration of strategic variables in the post entry/location game. The number of entrants in the entry/location game can be one or many. In the single entrant model, existing firms that have already entered the market are not allowed to respond to the decisions of the single entering firm by changing their entry/location decisions. The total number of entrants in the multiple entrant model can either be determined exogenously (fixed entry) or endogenously (free entry). In addition to the entry/location decision, some models consider other strategic decisions such as price, production quantity, and capacity in the post entry/location game. Table 1 shows the position of prior work using these criteria.

Table 1
A taxonomy of network-based competitive location models

Entry/Location game (No. of entrants)		Post entry/Location game (strategic variables)			
		None	Price	Quantity	Quantity & Capacity
Single entrant		ReVelle (1986)		Tobin and Friesz (1986); Friesz et al. (1989); Miller et al. (1991)	
Multiple entrants	Fixed no. of entrants (fixed entry)	Hansen and Thisse (1981); Wendell and McKelvey (1981); Hakimi (1983, 1986); Dobson and Karmarkar (1987); Bauer et al. (1993); Eiselt (1998)	Lederer and Thisse (1990)	Labbé and Hakimi (1991); Sarkar et al. (1997)	
	Unrestricted no. of entrants (free entry)			Special case of our model	Our model

1.2. The model

In this paper we investigate a three-stage competitive location game in which firms first choose their facility locations, second, production capacities, and then a production quantity for each market. There are a finite number of potential facility sites and a finite number of markets, which may or may not be co-located. These facility sites and markets are treated as nodes of a network. Fixed and variable costs of production vary with the location of the facility, and transportation costs depend on the distances between nodes. Each market has its own demand curve which relates the delivered price to the total available quantity at the market. In multi-product cases, production, transportation costs and demand curves may vary with the products produced. We analyze pure-strategy Nash equilibria for these games when the entry of firms is free, i.e., there are no barriers to entry other than the fixed costs.

Table 1 also shows the position of our model vis-à-vis the prior literature: First, our model is more general than models with a single entrant or a fixed number of entrants. For example, our model is an extension of Labbé and Hakimi (1991) and Sarkar et al. (1997). Labbé and Hakimi analyze a duopolistic game with zero fixed cost. Their model consists of two stages: a first stage for location and a second stage for production quantity decisions. In each stage, firms make decisions simultaneously. They show that if the quantity shipped by both firms at each market site is strictly positive, then there is at least one pure strategy Nash equilibrium (PNE) for the location game. An equilibrium may not exist when some market site is served by only a single firm. Sarkar et al. generalized Labbé and Hakimi's work to the game involving $n(\geqslant 2)$ firms. They find a similar condition for the existence of an equilibrium. We build on these works by considering free entry (simultaneous and sequential) with fixed cost and capacity limitations. These extensions lead to somewhat different results; for example, we show that under certain conditions an equilibrium for the simultaneous entry game exists even when market sites are served by a single firm.

Second, our model allows us to explicitly consider strategic variables such as production quantity and capacity. The market share in the models without these strategic variables is typically determined by simple allocation rules such as closest distance or probabilistic choice rules. In models with these strategic variables, demand is allocated by result of an *active* rivalry, that is, firms compete for the market demand with decision variables such as price, production quantity, and capacity. Lederer and Thisse (1990) apply Bertrand price competition for demand allocation, resulting in a natural monopoly in each market, that is, the firm that has the lowest cost serves the market. In the quantity competition model (so called Cournot model), multiple firms may serve a common market. [1] Our model belongs to the latter class. Also, the full version of our model considers production capacity as a strategic decision variable.

Third, our work extends solution concepts developed by Dobson and Karmarkar (1987). They study a duopolistic game in which two firms enter the industry sequentially. Transportation (or travel) costs are paid by the customers who choose the closest facility. The firm that enters first (the leader) determines the number and location of facilities. Thus, the leader will increase the number of facilities as long as each facility makes a positive profit. Then, the firm that enters later (the follower) is blockaded. The set of facilities opened by the leader is considered "stable" if each individual facility is profitable (*viability condition*) and any additional new facility opened by the follower will result in a loss (*survival condition*). We relate their notion of equilibrium, the so-called "stable set (SS)", to Nash equilibria in the game we study.

The overall formulation is a three-stage, multi-market, oligopolistic, spatial competition model on a network with heterogeneity in production costs. In the current analysis, we essentially model location in one layer of a supply chain. Plants with very heterogeneous structures can compete and survive in the same markets, though, of course, low cost plants capture higher market shares. These features qualitatively

[1] See Friedman (1977), Miller et al. (1991) for a discussion of the Cournot model.

match what can be observed in the sectors we mention. We present a single product model first, before extending to a multi-product case. These models generate the following results:

- In the first stage, we examine the existence of Nash equilibria in a simultaneous entry game. The existence for the sequential entry game is a classic result of Kuhn (1953). For the simultaneous entry game, we show that our game is related to so-called "congestion games" (Rosenthal, 1973) and provide sufficient conditions for the existence of Nash equilibria.
- We establish certain relationships between the Nash equilibrium sets of the first stage games and the SS of Dobson and Karmarkar (1987). We show that the Nash equilibrium sets of the sequential and simultaneous entry games are subsets of this SS.
- We show that capacity and production level decisions are collapsed into a single stage decision. We provide a polynomial time algorithm to find a unique Nash equilibrium in the capacity choice and production quantities. In the equilibrium, not all opened facilities supply to all markets. They select markets based on demand and variable costs incurred by capacity acquisition, production, and transportation.
- In the sequential entry game, we study whether first movers may enjoy a higher profit compared to later entrants. We show a condition where first movers may not always enjoy an advantage. This condition relates to a topological network structure where a "dancing phenomenon" occurs. [2] First mover advantage is guaranteed only when the Nash equilibrium set of the sequential entry game is a subset of the Nash equilibrium set of the simultaneous entry game.

This paper is organized as follows. In Section 2, we formulate the model. The capacity choice and production problems are analyzed first followed by the location problem. We then address the issue of first mover advantage. Section 3 concludes.

2. Problem formulation

The problem is formulated as a three-stage non-cooperative game. A three-stage model is appropriate here because location, capacity, and production decisions have different time horizons and flexibility. We suppose that each player has perfect information on the decisions made in the previous stage. That is, in the second (and third) stage, location (and capacity) decisions are given. While, this assumption is made for simplicity, it is not unreasonable, considering the time horizon of each decision. In each stage, firms make a decision to maximize their own profit.

In the first stage of the model, we consider two location games: in one game, firms enter simultaneously; in the other, firms enter sequentially. In the simultaneous game (G_T), each firm chooses a strategy without knowledge of its opponents' action. In the sequential game (G_S), later entrants have perfect information about their predecessors' decisions, so that the predecessors and later entrants have a Stackelberg leader and follower relationship.

In the second stage of the model, entering firms decide on their production capacities. Acquiring capacity may incur both fixed and variable costs. (Similar cost structure has been used by Karmarkar and Pitbladdo, 1993, 1994.) However, there is no reason that firms, as long as they decided to enter, would have zero capacity. Thus, any fixed costs of capacity acquisition are absorbed into the fixed costs of the previous stage, and only variable costs of capacity acquisition affect the capacity decision. Variable capacity costs are assumed to be site-specific rather than firm-specific.

[2] Teitz (1968) called it "dancing equilibrium". To avoid confusion with the game theoretic notion of equilibrium, we prefer not to use the word equilibrium. "Dancing" occurs when firms find it profitable to hop back and forth between locations so that no stable configuration exists (see Section 3.2).

In the third stage of the model, firms choose quantities to be produced and shipped to each market site. The total quantity available at each market site determines a market price via the given demand curve. Variable production and transportation costs, which are again assumed to be site-specific, are incurred at this stage.

In each stage, Nash equilibrium is used as a solution concept. We start from a single-product model, and later extend it to a multi-product case. In the latter case, demand and cost parameters also depend on the products. We first define the following notation for the single product model:

N^*	number of entrants,
N	potential number of firms in an industry, N is large enough such that $N \geqslant N^*$,
i	index for production sites, $i \in I = \{1, 2, \ldots, L\}$,
j	index for markets, $j \in J = \{1, 2, \ldots, M\}$,
r, s	indices for entrants, $r, s \in \{1, 2, \ldots N^*\}$,
x_r	location decision variable for firm r: i if the firm r opens its facility at site i; 0 if the firm does not enter the industry,
x	location decision vector, $x = (x_1, \ldots, x_N)$,
Z_i	set of indices of entrants who open facilities at site i, $Z_i = \{r \mid x_r = i\}$, Z_i may be empty,
Z	set of indices of entrants who open facilities, $Z = \bigcup_{i=1}^{L} Z_i = \{1, 2, \ldots, N^*\}$, $\lvert Z \rvert = N^*$,
K_r	production capacity of an entrant r,
K	capacity decision vector, $K = (K_1, \ldots, K_{N^*})$,
q_{ijr}	quantity produced by the entrant r at site i and shipped to market j,
$q_{ij\bullet}$	quantity produced at site i and shipped to market $j = \sum_{r \in Z_i} q_{ijr}$,
$q_{i\bullet\bullet}$	total quantity produced at site $i = \sum_{j=1}^{M} q_{ij\bullet}$,
$q_{\bullet j\bullet}$	total quantity available at market $j = \sum_{i=1}^{L} q_{ij\bullet}$,
Q	$(L \times M \times N)$ matrix of q_{ijr},
$p_j(q_{\bullet j\bullet})$	price at market j, a function of $q_{\bullet j\bullet}$,
p	price vector, $p = (p_1, \ldots, p_M)$,
v_i	variable cost of production at site i,
c_i	unit variable cost of acquiring capacity K_i at site i,
f_i	fixed cost of opening a facility at site i,
t_{ij}	variable cost of shipping product from production site i to market j,
$\pi_r(x, K, Q)$	net profit of the supplier r; $\pi_r(x, K, Q) = \sum_{j=1}^{M} [p_j(q_{\bullet j\bullet}) - v_i - t_{ij}] q_{ijr} - c_i K_r - f_i$, if $x_r \neq 0$ and $r \in Z_i$ for some i; else, $\pi_r(x, Q) = 0$.

2.1. Second and third stages: Capacitated Cournot quantity competition

We model capacity and production decisions of the entrants as a two-stage capacitated Cournot quantity game. Since location decisions are made in the first stage, x's and Z_i's are given in these stages. We first identify an equilibrium in a reduced problem of capacity and production level choices and show that this equilibrium is also a solution to the original problem. Thus, we will identify the capacity choices of entrants located at each production site i, the amounts shipped from production site i to market j, and the total quantity available at market j. The last will also determine the delivered price at market j.

Each of the facilities can ship the product to each of the market locations through the cheapest path. [3] The variable production cost v_i, transportation cost t_{ij}, and capacity acquiring cost c_i depend only on the

[3] The cheapest path, which is analogous to shortest path, can be obtained by the Floyd and Warshall algorithm within $O((L+M)^3)$ (see Lawler, 1976).

site so that firms at the same site have the same cost structure. The inverse demand function p_j is assumed to be linear so that $p_j(q_{\bullet j \bullet}) = a_j - b_j q_{\bullet j \bullet}$ where a_j and b_j are parameters representing the maximum demand level and price sensitivity at market j, respectively. Then net profit for the entrant $r \in Z_i$ is expressed as:

$$\pi_r(x, K, Q) = \sum_{j=1}^{M} \left[p_j(q_{\bullet j \bullet}) - v_i - t_{ij} \right] q_{ijr} - c_i K_r - f_i, \tag{1}$$

$$\text{subject to:} \quad \sum_{j=1}^{M} q_{ijr} \leqslant K_r.$$

Suppose that there exists an entrant with excess capacity. Then, the entrant can improve profit simply by reducing capacity. Thus, there is no reason to have excess capacity in equilibrium. That is, $\sum_{j=1}^{M} q_{ijr} = K_r$, and Eq. (1) is reduced to:

$$\pi_r(x, Q) = \sum_{j=1}^{M} \left[p_j(q_{\bullet j \bullet}) - v_i - t_{ij} \right] q_{ijr} - c_i \sum_{j=1}^{M} q_{ijr} - f_i. \tag{2}$$

This function is strictly concave in each supply quantity q_{ijr}. Since x is given at this stage, entrants maximize their profit by choosing their supply quantities. A Nash equilibrium for the reduced problem is then given by a set of q_{ijr} for each $r \in Z_i$ and each j such that $\partial \pi_r(x, Q)/\partial q_{ijr} = 0$, or

$$b_j q_{ijr} = \delta_{ij} - b_j q_{\bullet j \bullet}. \tag{3}$$

where $\delta_{ij} = a_j - v_i - c_i - t_{ij}$. In Eq. (3), q_{ijr} does not depend on the parameters and variables for the other markets. Thus, the firm's supply quantity decisions for the M markets can be made independently of each other. In other words, we can analyze each market separately. By adding equations of Eq. (3) for all entrants at a given market j, $b_j q_{\bullet j \bullet} = \sum_{k=1}^{L} |Z_k| \delta_{kj} - b_j \sum_{k=1}^{L} |Z_k| q_{\bullet j \bullet}$, or

$$q_{\bullet j \bullet} = \frac{\sum_{k=1}^{L} |Z_k| \delta_{kj}}{b_j(|Z| + 1)}. \tag{4}$$

From Eqs. (3) and (4),

$$q_{ijr} = \frac{(|Z| + 1)\delta_{ij} - \sum_{k=1}^{L} |Z_k| \delta_{kj}}{b_j(|Z| + 1)}. \tag{5}$$

In deriving Eq. (5), we neglected the condition that supply quantities must be non-negative. The non-negativity of supply quantities is violated when there exists $r \in Z_i$ such that $\delta_{ij} < \left(\sum_{k=1}^{L} |Z_k| \delta_{kj}\right)/(|Z| + 1)$.

Lemma 1 states the existence of a set of non-negative supply quantities q^*_{ijr}. Before the lemma is stated, we define a new index to rank the order of the production sites. For each market j, we create a new index set where the order of the elements is decreasing in δ_{ij}. Let $l_j(i)$ be the order of production site i with respect to market j in this new index. Then we have:

Lemma 1. *For a market j, suppose that there exists some production site i^* such that $|Z_{i^*}| > 0$ and $\delta_{i^* j} \leqslant \left(\sum_{k=1}^{L} |Z_k| \delta_{kj}\right)/(|Z| + 1)$. Then, there exists a unique rank order $l^*_j < L$ in which production sites whose rank order are higher produce positive quantities and whose rank order are lower produce zero*:

$$q^*_{ijr} = \frac{\left(\sum_{l_j=1}^{l^*_j} |Z_{l_j}| + 1\right)\delta_{ij} - \sum_{l_j=1}^{l^*_j} |Z_{l_j}| \delta_{l_j j}}{b_j\left(\sum_{l_j=1}^{l^*_j} |Z_{l_j}| + 1\right)} \quad \mathit{if} \ \ l_j(i) \leqslant l^*_j < L, \mathit{and} \tag{6}$$

$$q^*_{ijr} = 0, \quad \mathit{if} \ \ l_j(i) > l^*_j,$$

where $r \in Z_{l_j} \neq \phi$.

The proof of Lemma 1 is provided in the appendix. Similar results were presented by Anderson and Neven (1990). They showed the existence of a unique Nash equilibrium for the uncapacitated Cournot game. If $c_i = 0$ for all i, our result corresponds to Anderson and Neven (1990). However, our original problem is a capacitated Cournot game. We obtain the solution to our problem by showing that supply quantities in Lemma 1 generate an equilibrium for the original capacitated problem.

Proposition 1. *The q^*_{ijr}'s in Eq. (6) and $K^*_r = \sum_{j=1}^{M} q^*_{ijr}$ are a Nash equilibrium for the two-stage capacitated Cournot quantity game. (The proof is provided in the appendix.)*

Proposition 1 provides a way to obtain a Nash equilibrium for the capacitated Cournot game when the variable costs are not uniform. In the proof of Lemma 1, l^*_j is defined as:

$$l^*_j = \max\left\{ k_j \left| \delta_{k_j j} > \frac{\left(\sum_{l_j=1}^{k_j} |Z_{l_j}|\delta_{l_j j}\right)}{\left(\sum_{l_j=1}^{k_j} |Z_{l_j}| + 1\right)} \right.\right\}.$$

l^*_j can be produced by simply ranking the production sites and searching for the k_j which satisfies the given condition. Thus, Lemma 1 and Proposition 1 enable us to construct an exact algorithm to obtain the set of capacity choice K_r, supply quantities q_{ijr}, price p, and therefore profit π_r for each entrant r within a polynomial-time bound. (The algorithm and its computational complexity are described in the appendix.)

We call entrants who ship positive quantity to a market, *active* in that market. As long as the condition of Lemma 1 is satisfied in some market, there exists a firm that does not supply to all markets. Thus, the number of active firms in some market is less than the total number of entrants N^*. Corollary 1 states this result formally.

Corollary 1. *Define N^*_j as the number of active entrants supplying to market j. For a market j, suppose that there exists some production site i such that $|Z_i| > 0$ and $\delta_{ij} < \left(\sum_{k=1}^{L} |Z_k|\delta_{kj}\right)/(|Z|+1)$. Then, in a Nash equilibrium the total number of active facilities or entrants supplying to market j, $N^*_j \equiv \sum_{l_j=1}^{l^*_j} |Z_{l_j}| < |Z|$ (i.e. the total number entrants).*

Corollary 1 implies that firms do not supply to all markets, resulting in a phenomenon of *localization of supply*. For given Z_i's, N^*_j is a function of l^*_j, which is a function of δ_{ij}. Thus, N^*_j is determined by δ_{ij}'s. An increase in the maximum demand level a_j in market j or a decrease in capacity acquiring cost c_i, production cost v_i and transportation cost t_{ij} at site i will lead to an increase in δ_{ij} and hence the number of markets to which a firm can supply.

These results may be easily extended to a multi-product case. Suppose that firms produce multiple products rather than a single commodity. It is assumed that the demand of each product is independent of other products, while each product of a firm competes for a common resource, that is, production capacity. Then the net profit of entrant $r \in Z_i$ is expressed as:

$$\pi_r(x, K, Q) = \sum_{h=1}^{H}\sum_{j=1}^{M}\left[p^h_j(q^h_{\bullet j \bullet}) - v^h_i - t^h_{ij}\right] q^h_{ijr} - c_i K_r - f_i \tag{7a}$$

$$\text{subject to:} \quad \sum_{h=1}^{H}\sum_{j=1}^{M} q^h_{ijr} \leqslant K_r,$$

where $h(=1,\ldots,H)$ is an index for products produced. With the same reasoning as in the single product case, $\sum_{h=1}^{H}\sum_{j=1}^{M} q^h_{ijr} = K_r$ and Eq. (7a) reduces to:

$$\pi_r(x, Q) = \sum_{h=1}^{H} \sum_{j=1}^{M} \left[p_j^h(q_{\bullet j \bullet}^h) - v_i^h - t_{ij}^h - c_i\right] q_{ijr}^h - f_i \tag{7b}$$

From the first-order condition of Eq. (7b), the firm's supply quantity decisions for the H products in M markets can be made independently of each other. Thus, similar results are obtained: i.e., supply quantities obtained using Eq. (6) for each product generate overall capacities of entrants and shipments of each product.

2.2. First stage: Location decision

In this subsection, we investigate Nash equilibria for two location decision games: the simultaneous entry game (G_T) and the sequential entry game (G_S). Also, we define "SS" as introduced by Dobson and Karmarkar (1987) and relate them to Nash equilibria. Finally, we discuss the existence of pure strategy Nash equilibria for both games.

In this stage, each firm r maximizes $\pi_r(x) = \pi_r(x, K^*(x), Q^*(x))$ by selecting x_r, considering the equilibrium capacity choice $K^*(x)$ and supply quantity $Q^*(x)$ of the following stages. We use PNE as the solution concept. At equilibrium, no firm (player) can benefit from a unilateral change in its location decision. That is, a location vector x^* is a PNE in the first stage, iff $\pi_r(x_r^*, x_{-r}^*) \geqslant \pi_r(x_r, x_{-r}^*)$ for all x_r and all r, where x_{-r} denotes a strategy profile of all firms but r. Since the equilibria of the second and third stages are Nash, the equilibrium obtained in the first stage is Subgame Perfect (Selten, 1975).

Dobson and Karmarkar (1987) introduce the notion of a "SS". [4] According to them, a set of facility locations is regarded as *stable*, if and only if the entrants make a profit (*viability*) and potential entrants cannot find any location where their profit after entry is positive (*survival*). In Dobson and Karmarkar's problem formulation, the stability concept describes the state of an industry configuration rather than active decisions of firms. The vector of the number of firms at each production site is defined as an *occupancy vector* and expressed as $(|Z_1|, \ldots, |Z_L|)$. [5] Then a SS is defined formally as follows:

Definition 1 (*Stable set* (*SS*) *in the first stage*)**.** The occupancy vector $(|Z_1|, \ldots, |Z_L|)$ is an element of a SS, iff (i) for all r such that $r \in Z$, $\pi_r \geqslant 0$ (*viability*), and (ii) for any s such that $s \notin Z$, $\pi_s < 0$, if s enters (*survival*).

Before we investigate the relationship between SSs and Nash equilibria, we define a "restricted game" in which firms have pre-assigned locations but must simultaneously decide whether to enter or not. In this game, the decision of each firm can be represented by a binary value: 0 if a firm decides not to enter; 1 if a firm decides to enter. We denote this game, G_{01}. Then we can show that pure strategy Nash equilibria of G_T, G_S, and G_{01} always satisfy the conditions for stability and that the set of pure strategy Nash equilibria of G_{01} is equivalent to the SS. (The proof is provided in the appendix.)

Proposition 2. (i) $SS \supset PNE(G_T)$; (ii) $SS \supset PNE(G_S)$; (iii) $SS = PNE(G_{01})$.

Proposition 2 provides us with a different way to understand Nash equilibrium in competitive location games. The proposition indicates that the Nash equilibria must satisfy viability and survival conditions; otherwise, firms have incentives to change their action. For example, if the profit of an entrant is negative,

[4] They define several variants of SS according to the context of the problem. We refer to the SS defined by their variants with the properties of Weak survival, Restricted entry, and Independent viability.

[5] Unlike in the second stage, Z_i is a variable in the first stage.

which is a violation of viability, then the entrant will move out of the industry. If a potential entrant can make a positive profit by establishing a facility, which is a violation of the survival condition, then the firm enters the industry. These results explain why the later entrants are blockaded, even though co-location at each production site is not prohibited.

Proposition 2 also gives us a way to produce PNE(G_T) and PNE(G_S). For example, part (iii) implies that stability does not guarantee the Nash equilibria of G_T. This is because each entrant may have an incentive to move to other production sites, even though the stability condition is satisfied. Thus, PNE(G_T) is identified by testing each element of the SS in the following way. Suppose an occupancy vector $(|Z_1|, \dots, |Z_L|)$ is stable. We reduce $|Z_i|$ by one and test whether the entrant leaving site i can be better off at any other sites. This procedure is repeated for all sites $i = 1, \dots L$. If no entrant can be better off during this procedure, then this occupancy vector corresponds to a Nash equilibrium. (An algorithm is provided in the appendix.)

The search for PNE(G_S) is narrowed by Proposition 2. First, it provides the maximum number of players in an industry for the free entry model so that the depth of the game tree is determined a priori. Once the depth of the game tree is restricted, one needs only to look at the branches generated by the SS rather than the whole game tree (which is given in Fig. 3). This restricted search implies that players may foresee the ends, but only the limited ends. The algorithm may be sketched as follows:

Step 1. Obtain elements of the SS.
Step 2. Find PNE(G_S) among the elements of the SS.

When identifying PNE(G_T) and PNE(G_S), we need to have the SS on hand. SSs can be generated by search algorithms such as genetic algorithms. Rhim (1997) presents the implementation of the genetic algorithms. Applying non-binary representation, chromosomes to be evolved are occupancy vectors. Genetic algorithms seek balance between population diversity and selective pressure. Unlike optimization problems, generating almost all elements of the SS within a reasonable time bound is critical to identifying right equilibrium points. Therefore, population diversity is emphasized more than selective pressure. In the following example, we illustrate the computation of Nash equilibria and discuss properties of Proposition 2.

Example 1. (i) *Stable set*: Suppose that costs and inverse demand curves are as given in Fig. 1. Let $(f_1, f_2, f_3) = (35, 17, 7)$. For simplicity, suppose that $c_i = 0$ for all i. In order to enumerate the occupancy vectors, we need an upper bound of the potential number of entrants at each site. The bound is obtained by

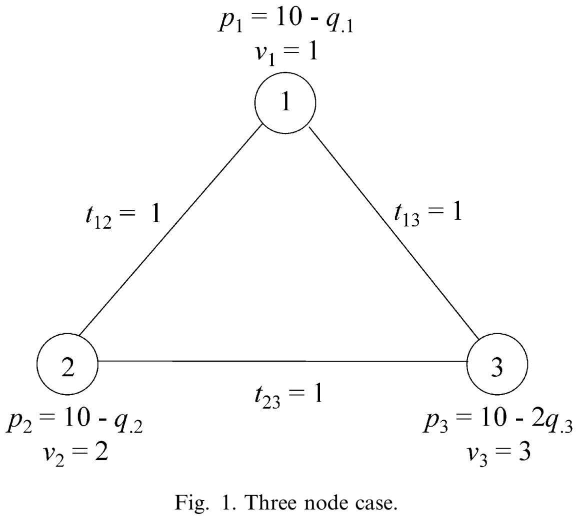

Fig. 1. Three node case.

Table 2
Occupancy-payoff table of the network in Fig. 2

Occupancy vector	π_{Z_1}	π_{Z_2}	π_{Z_3}	Occupancy vector	π_{Z_1}	π_{Z_2}	π_{Z_3}
(0,0,0)	0	0	0	**(1,0,0)**	**9.25**	**0**	**0**
(0,0,1)	0	0	17.12	(1,0,1)	−3.39	0	−2.22
(0,0,2)	**0**	**0**	**3.72**	(1,0,2)	−8.81	0	−4.31
(0,1,0)	0	17.38	0	(1,1,0)	−9.94	−5.11	0
(0,1,1)	**0**	**3.94**	**0.28**	(1,1,1)	−13.38	−7.38	−5.38
(0,1,2)	0	−1.41	−2.91	(1,1,2)	−15.26	−8.56	−5.96

increasing the number of firms at a production site until the total profit of each firm located at the site becomes negative, while keeping the other sites unoccupied. The resulting upper bound is (1, 1, 2). Then, we enumerate the occupancy vectors up to the upper bound. Profits of individual firms at each production site i, π_{Zi}'s according to the occupancy vector $(|Z_1|, \dots, |Z_L|)$ are summarized in Table 2, the *occupancy-payoff table*. This table allows us to identify the SS, which is $\{(0,0,2),(0,1,1),(1,0,0)\}$. Note that the number of entrants in each element of SSs can be different: i.e., $1 \leqslant N^* \leqslant 2$.

(ii) *PNE*(G_T): We can represent this game with a two-dimensional payoff matrix as shown in Fig. 2. The strategy set of individual firms is $\{0,1,2,3\}$, where 0 represents no-entry decision. The payoff matrix can be constructed from Table 2 except for some diagonal elements which are always dominated by a no-entry decision. While (1,0), (0,1), (2,3), (3,2), and (3,3) are possible strategy pairs from the SS, only (2,3) and (3,2) belong to PNE(G_T). This result illustrates that SS $\supset$ PNE(G_T). On the other hand, Nash equilibria can be obtained directly from the SS. Among three elements of the SS, only (0,1,1) satisfies the condition of moving nowhere. That is, no entrant can be better off by moving to other sites. Therefore, (2,3) and (3,2) is constructed as PNE(G_T).

(iii) *PNE*(G_S): The SS limits the number of entrants to at most two so that we can construct a game tree to identify PNE(G_S), as in Fig. 3. The branches are strategy profiles of the entrants in the first stage, while the numbers at the end of the leaves are the payoffs of firm 1 and firm 2 after the third stage game. Backward induction starts from the decision of firm 2. For each subgame, firm 2 selects the best strategy by comparing the payoffs. Then, the game is reduced to a profit-maximization problem of firm 1. Since selecting site 1 is the best strategy for firm 1, (1,0) is identified as PNE(G_S). In this procedure, the relationship established in Proposition 2 enables us to restrict the search only to the solid branches. The size of the tree to be searched is reduced by 75% in this example.

The existence of PNE of G_S is guaranteed by backward induction since each information set of the game tree is a singleton (Kuhn, 1953). Non-emptiness of the SS is guaranteed by Proposition 2 and the existence of PNE of G_S. However, a PNE of G_T may not exist, as shown in Example 2.

Player II

Player I		0	1	2	3
	0	(0, 0)	**(0, 9.25)**	(0, 17.38)	(0, 17.12)
	1	**(9.25, 0)**	(-, -)	(-9.94, -5.11)	(3.39, -2.22)
	2	(17.38, 0)	(-5.11, -9.94)	(-, -)	**(3.94, 0.28)***
	3	(17.12, 0)	(-2.22, 3.39)	**(0.28, 3.94)***	**(3.72, 3.72)**

Fig. 2. Two-dimensional payoff matrix of three node case.

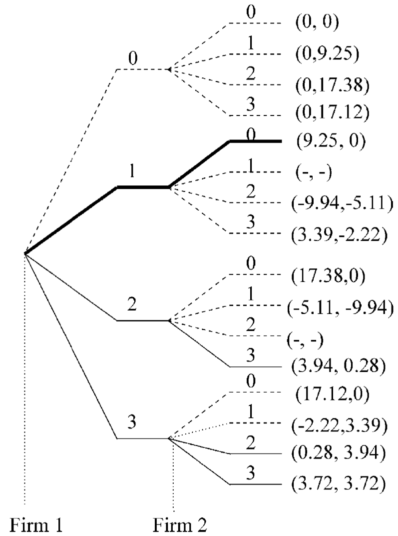

Fig. 3. A game tree to identify PNE(Gs).

Example 2. Suppose that costs and inverse demand curves are as given in Fig. 4. Possible facility sites are limited to nodes 1, 3, 5 because of the high fixed cost at other nodes. Markets exist only at nodes 2, 4, 6. The occupancy-payoff table is provided in Table 3. In this example, (1,0,1,0,1,0) is not viable. Also, cost and demand parameters do not allow more than one firm to open facilities at the same site. Thus, $\{(1,0,1,0,0,0),(1,0,0,0,1,0),(0,0,1,0,1,0)\}$ is the SS.

Since the number of entrants in each occupancy vector of the SS is not greater than two, we can represent this game with a two-dimensional payoff matrix as shown in Fig. 5. We do not have to consider the no-entry decision since it is dominated by the other entry decisions. Fig. 5 shows that a pure strategy Nash equilibrium does not exist. This can happen when the variable production and transportation costs have a special pattern, giving firms incentive to deviate from the current decision. For example, if player 1 enters site 1, player 2 chooses site 3, since site 3 has a cost advantage over site 1. Then player 1 will move to site 5, which will make player 2 move to site 1. Thus, the two players will hop around the sites endlessly. This phenomenon was called "dancing" by Teitz (1968). It was also observed by Labbé and Hakimi (1991), who dealt with the duopoly case *without* fixed cost. However, this example shows that this phenomenon also happens in the free entry problem with fixed cost.

Example 2 shows that PNE of G_T may not exist in general. However, we show below that if the profit function of the firm depends only on the number of other firms choosing the same strategy profile, that is, $\pi_r(x) = f_i(|Z_i|)$ for any player $r \in Z_i$, the existence of PNE of G_T can be guaranteed. Before we present the result, we need to define the notion of *generic* game. A game G_T is called generic if $f_i(|Z_i|) \neq f_{i'}(|Z_{i'}|)$ for every $i \neq i'$, and $1 \leqslant |Z_i|, |Z_{i'} \leqslant N^*$. (See Milchtaich, 1996b.)

Proposition 3. (a) *Suppose that there exist a production site* $\hat{i}_j$ *at every market* j *such that*; (i) $v_{\hat{i}_j} + t_{\hat{i}_j j} + c_{\hat{i}_j} < v_i + t_{ij} + c_i$ *for any* $i \neq \hat{i}_j$; (ii) $2(v_i + t_{ij} + c_i) - (v_{\hat{i}_j} + t_{\hat{i}_j j} + c_{\hat{i}_j}) > a_j$ *for any* $i \neq \hat{i}_j$; (iii) $\sum_{k \in S_{\hat{i}_j}} (a_k - v_{\hat{i}_j} - t_{\hat{i}_j k} - c_{\hat{i}_j})^2 / 4b_k > f_{\hat{i}_j}$, *where* $S_{\hat{i}_j} \equiv \{k | v_{\hat{i}_j} + t_{\hat{i}_j k} + c_{\hat{i}_j} < v_i + t_{ik} + c_i$ *for any* $i \neq \hat{i}_j\}$. *Then,* $\pi_r(x) = f_i(|Z_i|)$ *for any player* $r \in Z_i$, *and there exists a PNE of* G_T.

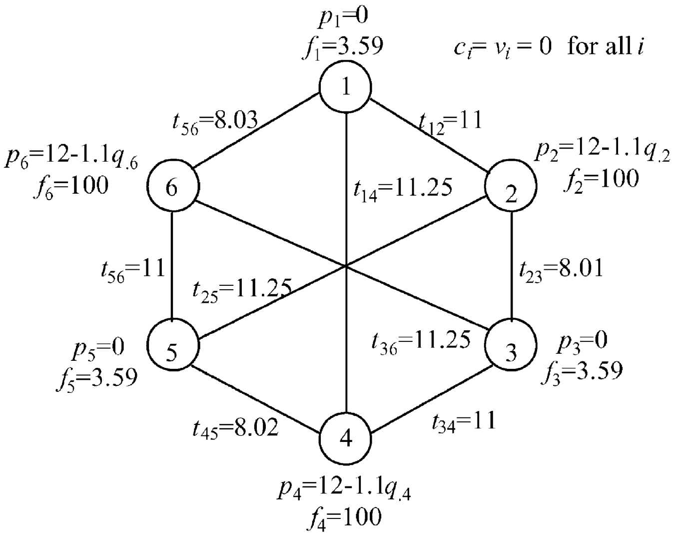

Fig. 4. A case for dancing phenomenon.

Table 3
Occupancy-payoff table of the network in Fig. 4

Occupancy vector	π_{Z_1}	π_{Z_2}	π_{Z_3}	π_{Z_4}	π_{Z_5}	π_{Z_6}
(1,0,1,0,0,0)	0.02	0	0.19	0	0	0
(1,0,0,0,1,0)	0.15	0	0	0	0.04	0
(0,0,1,0,1,0)	0	0	0.05	0	0.17	0
(1,0,1,0,1,0)	−0.01	0	0.03	0	0.01	0

		Player II 1	Player II 3	Player II 5
Player I	1	(-1.84, -1.84)	(0.02, 0.19)	(0.15, 0.04)
	3	(0.19, 0.02)	(-1.82, -1.82)	(0.05, 0.17)
	5	(0.04, 0.15)	(0.17, 0.05)	(-1.83, -1.83)

Fig. 5. Two-dimensional payoff matrix of the dancing case.

(b) *Suppose that* $v_i + t_{ij} + c_i = v_{i'} + t_{i'j} + c_{i'}$ *for all* i, i' *at every market* j *and that there exists* i^* *such that* $f_{i^*} < f_i$ *for all* i *and* $(a_j - v_{i^*} - t_{i^*} - c_{i^*})^2/4b_j > f_{i^*}$. *Then there always exists a PNE of* G_T. *Furthermore, all entrants are located at the site* i^*.

(c) *Let* G_S *be a corresponding sequential move game of* G_T. *If part* (*a*) *holds and* G_T *is generic*, $\mathrm{PNE}(G_S) \subset \mathrm{PNE}(G_T)$. *Under the conditions of part* (b), *PNE*$(G_S) \subset$ *PNE*(G_T) *always holds.*

Part (a) of Proposition 3 maintains that if every market is served only by one production site and each production site is profitable enough to accommodate at least one firm, then there exists at least one PNE of G_T. This condition generates circumstances similar to Bertrand price competition, since most of the pricing games result in a natural monopoly in each market. The proof is based on general the results of congestion games, which are introduced by Rosenthal (1973) and generalized by Monderer and Shapley (1991), Milchtaich (1996a). Congestion games are a class of games where the payoff of a player is a non-increasing function of the number of players who select the same strategy profile. We prove existence by showing that our model is a special case of congestion games. Thus, under the given conditions of (a), firms at one production site do not depend on the strategy profiles of the firms at the other production sites. That is, firms located at one production site construct their own territory consisting of exclusively served markets.

Part (b), unlike part (a), addresses the case when the total variable costs are indifferent. In this equilibrium, identical quantities are supplied from individual entrants regardless of their site decisions. Then the gross contributions of productions before fixed costs are again identical. Thus entrants eventually select the site with the cheapest fixed cost. A special case of part (b) occurs when only a single market exists. This case might be interpreted as a metropolitan market surrounded by possible production sites in the ex-urban area. The proposition suggests that facilities concentrate in the site with the cheapest fixed cost when they cannot take advantage of variable costs. Conversely, firms are dispersed around the metropolitan market only if there are significant trade-offs between variable costs and fixed costs.

Part (c) addresses the relationship between simultaneous and sequential games under the given conditions. While the proof under the conditions of part (b) is obvious, that of part (a) depends on results by Milchtaich (1996a,b), who shows the relationship between simultaneous and sequential move congestion games. By this relationship, the first mover's advantage over its followers is guaranteed. This will be addressed in the next section.

Labbé and Hakimi (1991) present a condition for the duopolistic game, and Sarkar et al. (1997) generalize it to the n-person game. The condition is such that the total variable costs must be sufficiently small for any location, implying that every entrant should serve every market. Our condition, found in part (a), is somewhat different from theirs. Under our condition, firms localize their territories. In other words, the total market is regionalized and no interaction occurs between regional markets.

2.3. First mover advantage

In the sequential decision game G_S, firms who make location decisions earlier (or first movers) may or may not have an advantage over their followers. In the following proposition, we show when first mover advantage is guaranteed. By first mover advantage, we mean that the profit of the first mover is greater than or equal to that of its followers. Proofs are provided in the appendix.

Proposition 4. *In the sequential entry game G_S, first mover advantage is ensured if PNE of G_T exists and $PNE(G_S) \subset PNE(G_T)$.*

Proposition 4 states that firms entering the market early will be better off than their followers only if certain conditions are satisfied. The conditions can be clarified by examining Example 2. Suppose that firms enter the industry sequentially, while other problem settings are unchanged. Then, the follower observes the move of the first entrant and deviates from the decision which endows the first mover with a cost advantage in the example. However, if a Nash equilibrium exists in the simultaneous entry (for example, the dancing phenomenon does not occur), and if the Nash equilibrium of the sequential game is a subset of the Nash equilibria for the simultaneous game, then the follower's best response is to select the site which endows the first mover with a cost advantage. Therefore, the first mover is better off than the follower.

The phenomenon that the strong first mover advantage does not exist was called the "first entry paradox" by Ghosh and Buchanan (1988). They discuss the first entry paradox with a duopolistic location model in a linear market, and observe the relationship between the first entry paradox and the non-existence of a Nash equilibrium. Our result in Proposition 4 is a generalization of their result, since we deal with an oligopolistic model on a network.

3. Conclusion

Our eventual aim in this research is to develop methods for the support of plant location decisions in process industries. However, the underlying problem is of some technical difficulty, and the present state-of-the-art in modeling such problems is not very advanced. Our approach in this paper is to develop the modeling approach to a level that is rich enough to tackle realistic problems.

The models analyzed in this paper provide qualitative insights, as well as progress towards numerical techniques for analyzing location decisions in a competitive setting. An important issue in terms of qualitative results is the analysis of the competitive reach of plants. Given a set of locations, the Cournot model, unlike the Bertrand competition, shows that multiple entrants can compete in a given market. However, the ability and willingness to compete diminish as the cost of supplying a market increases.

From a strategic perspective, the analysis shows a problem for the case of simultaneous entry, which appears as the "dancing" phenomenon and the absence of a guaranteed equilibrium. Intuitively, what is happening here is that without the pre-commitment that occurs in sequential entry, the competitive situation is unstable. On the other hand, with sequential entry, there may be no advantage to early entry. This situation is not unlike the popular "rock-paper-scissors" game. In the location case, the lack of advantage for the first mover is created by the possibility of later entrants occupying locations that can "surround" the early entrant.

In some situations, the nature of location decisions can be driven towards simultaneity of entry. For example, some countries require licenses to establish plants and locations for certain industries. These licenses are often granted over a very short time period. The situation then resembles simultaneous entry. Furthermore, the short time available for decision making and the lack of information about competitors' intentions often provide insufficient time to do a thorough analysis. The resulting "gold rush" can exacerbate the situation. Examples can be found in the licensing policies practiced in India for many years. There, the result has been a plethora of so-called "sick" firms which, having committed resources in the initial entry period, are stuck with poor locations and no alternatives for exit.

In terms of numerical solutions, the second and third stage consequences for capacity, quantity, and price, given the locations, can be analyzed quite easily. However, the entry decision remains a difficult one. At this stage, it is feasible to analyze small problems which involve a few (less than 25) location choices depending on the number of players. This is in fact still quite useful; for example, a firm may want to examine the location of new plant, given the existence of some plants and the possibility of entry of competitors. For larger problems, we are able to generate "good" strategies and examples of patterns of competitive equilibrium. However, determining complete solutions to large problems is still out of our reach.

In our model, we assumed single-stage production. However, parts fabrication and final product assembly can be performed at different sites. Corbett and Karmarkar (2001) addressed competition in a multi-tier supply chain without spatial consideration. Allowing multiple-stage production in our model is a challenging extension and left to the future research.

Appendix A

Proof of Lemma 1. (i) Existence: Define l_j^* as

$$l_j^* = \max\left\{k_j \middle| \delta_{k_j j} > \left(\sum_{l_j=1}^{k_j} |Z_{l_j}|\delta_{l_j j}\right) \middle/ \left(\sum_{l_j=1}^{k_j} |Z_{l_j}| + 1\right)\right\}. \tag{A.1}$$

Produce Eq. (3) only using equations for sites $l_j(i) \leqslant l_j^*$ for each j. Then, for each market j:

$$q_{ijr}^* = \frac{\left(\sum_{l_j=1}^{l_j^*} |Z_{l_j}| + 1\right)\delta_{ij} - \sum_{l_j=1}^{l_j^*} |Z_{l_j}|\delta_{l_j j}}{b_j\left(\sum_{l_j=1}^{l_j^*} |Z_{l_j}| + 1\right)} \quad \text{if } l_j(i) \leqslant l_j^*;\ q_{ijr}^* = 0, \text{ if } l_j(i) > l_j \tag{A.2}$$

where $r \in Z_i \neq \phi$. The profit function is concave in each supply quantity, and Eq. (A.2) satisfies Kuhn–Tucker conditions simultaneously. Thus, these supply quantities are a Nash equilibrium. In this equilibrium, $l_j^* < L$ and $q_{ijr}^* \geqslant 0$, which proves existence. (ii) Uniqueness: Let $q_{k_j jr}(k_j)$ be the supply quantity from an entrant r such that $r \in Z_{k_j}$ where $k_j - 1$ is a specific value of index l_j. Suppose that site $(k_j - 1)$ is currently being considered as l_j^*. We show uniqueness by proving that $q_{k_j jr}(k_j)$, once it becomes negative, implies that $q_{(k_j+1)jr}(k_j + 1)$ cannot be positive. At each site, multiple entrants can exist, but their δ_{ij}'s are homogeneous. Hence, we consider only the last entrant of each site. Without loss of generality, we assume that $|Z_{k_j}|$, $|Z_{k_j+1}| > 0$. Let us denote the last entrants of the k_jth and $(k_j + 1)$th sites by r and s, respectively. Then,

$$q_{k_j jr}(k_j) - q_{(k_j+1)js}(k_j + 1) = \frac{z(k_j)z(k_j+1)(\delta_{k_j j} - \delta_{k_j+1 j}) + |Z_{k_j+1}|\left(z(k_j+1)\delta_{k_j+1 j} - \sum_{l_j=1}^{k_j+1} |Z_{l_j}|\delta_{l_j j}\right)}{z(k_j)z(k_j+1)b_j} \tag{A.3}$$

where $z(k_j) = \left(\sum_{l_j=1}^{k_j} |Z_{l_j}| + 1\right)$. Now, suppose $q_{k_j+1 jr}(k_j + 1)$ has a positive value, while $q_{k_j jr}(k_j)$ has a non-positive value. Then, $\delta_{k_j j} > \delta_{k_j+1 j}$, and from Eq. (A.1), $z(k_j+1)\delta_{k_j+1 j} - \sum_{l_j=1}^{k_j+1} |Z_{l_j}|\delta_{l_j j} > 0$. Thus, $q_{k_j jr}(k_j) - q_{(k_j+1)js}(k_j + 1) > 0$, which is a contradiction. Therefore, once $q_{k_j jr}(k_j)$ has a non-positive value, then any $q_{k_j+1 jr}(k_j + 1)$, cannot have a positive value, which proves the uniqueness of l_j^*. $\square$

Proof of Proposition 1. (i) First, we show that $K_r^* = \sum_{j=1}^{M} q_{ijr}^*$ is a Nash equilibrium for the capacity decision stage of the original game. Total differentiation of $K_r = \sum_{j=1}^{M} q_{ijr}$ generates $\mathrm{d}K_r = \sum_{j=1}^{M} \mathrm{d}q_{ijr}$ or $\sum_{j=1}^{M} (\mathrm{d}q_{ijr}/\mathrm{d}K_r) = 1$. Then,

$$\begin{aligned}\frac{\mathrm{d}\pi_r(K,Q)}{\mathrm{d}K_r} &= \sum_{j=1}^{M} \frac{\partial\pi_r(K,Q)}{\partial q_{ijr}} \cdot \frac{\mathrm{d}q_{ijr}}{\mathrm{d}K_r} + \frac{\partial\pi_r(K,Q)}{\partial K_r} = \sum_{j=1}^{M} \frac{\partial\pi_r(K,Q)}{\partial q_{ijr}} \cdot \frac{\mathrm{d}q_{ijr}}{\mathrm{d}K_r} - c_i \\ &= \sum_{j=1}^{M} \frac{\partial\pi_r(K,Q)}{\partial q_{ijr}} \cdot \frac{\mathrm{d}q_{ijr}}{\mathrm{d}K_r} - c_i \sum_{j=1}^{M} \frac{\mathrm{d}q_{ijr}}{\mathrm{d}K_r} = \sum_{j=1}^{M} \left[\frac{\partial\pi_r(K,Q)}{\partial q_{ijr}} - c_i\right] \frac{\mathrm{d}q_{ijr}}{\mathrm{d}K_r}\end{aligned}$$

From the first order condition of Eq. (2) and non-negativity condition, $q_{ijr}\left((\partial\pi_r(K,Q)/\partial q_{ijr}) - c_i\right) = 0$.

On the boundary points, i.e, if $(\partial\pi_r(K,Q)/\partial q_{ijr}) - c_i < 0$ holds, $\mathrm{d}q_{ijr}/\mathrm{d}K_r = 0$. Therefore, $\mathrm{d}\pi_r(K,Q)/\mathrm{d}K_r = 0$. (ii) Second, we show that q_{ijr}^*'s are a Nash equilibrium of the original problem, using contradiction. In the production quantity game, capacity acquiring costs are sunk. From Eq. (5), reduction in δ_{ij} generates an incentive to increase q_{ijr}. Suppose that q_{ijr}^*'s are not a Nash equilibrium, that is, there exists a firm r and market j such that the firm is better off by increasing q_{ijr}. Then, by increasing $K_r = \sum_{j=1}^{M} q_{ijr}$ s, firm r can be better off, which is a contradiction. $\square$

Proof of Proposition 2

(i) SS $\supset$ PNE(G_T): Suppose that there exists x^* such that $x^* \notin$ SS. Then, there are two cases: x^* is not viable, or is viable but not stable in the sense of survival.

Case 1: x^* is not viable: It implies that there exists a firm r such that $x_r^* \neq 0$, and $\pi_r(x_r^*, x_{-r}^*) < 0$. Since $\pi_r(0, x_{-r}^*) = 0 > \pi_r(x_r^*, x_{-r}^*)$, $x^* = (x_r^*, x_{-r}^*)$ is not PNE of G_T.

Case 2: x^* is viable, but not stable in the sense of survival: It implies that there exists a firm r such that $x_r^* = 0$, and $\pi_r(x_r, x_{-r}^*) \geqslant \pi_r(x_r^*, x_{-r}^*)$. Thus $x^* = (x_r^*, x_{-r}^*)$ is not a PNE of G_T. $\square$

Therefore, in both cases, x^* cannot be a PNE of G_T.

(ii) $SS \supset PNE(G_S)$: The proof is similar to that of (i).

(iii) $SS = PNE(G_{01})$

(a) $SS \subset PNE(G_{01})$: For the proof, we define new variables y_r such that $y_r = 1$ if the firm opens its facility at a preassigned site or $y_r = 0$. Let $Y = (y_1, \ldots, y_N)$. Suppose that there exists Y^* such that $Y^* \notin PNE(G_{01})$. Then there exists a firm r such that

$$\pi_r(y_r, y^*_{-r}) > \pi_r(y^*_r, y^*_{-r}) \tag{A.4}$$

Since y^*_r can have only two values, we examine the following two cases.

Case 1: $y^*_r = 0$, $y_r = 1$: Then Eq. (A.4) implies that $Y^* = (y^*_r, y^*_{-r})$ is not stable in the sense of survival.

Case 2: $y^*_i = 1$, $y_i = 0$: From Eq. (A.4), $\pi_r(0, y^*_{-r}) = 0 > \pi_r(1, y^*_{-r})$. Thus $Y^* = (y^*_r, y^*_{-r})$ is not viable.

Therefore, in both case, $Y^* = (y^*_r, y^*_{-r})$ does not belong to SS.

(b) $SS \supset PNE(G_{01})$: The proof is similar to that of (i). □

Proof of Proposition 3. (a) We prove existence by showing that our problem is a special case of congestion games. Congestion games are defined as follows: There are n players who share a common set $I = \{1, 2, \ldots, L\}$ of pure strategies and payoff functions π_i's; the payoffs π_i's are monotonically non-increasing functions of the number of players who select the ith strategy. Rosenthal (1973) defined such games and proved the existence of a PNE.

In order to transform our free entry model into a fixed entry model, we add an artificial production site to the original sites. The variable and fixed production costs in the artificial site are zero and transportation costs to the markets are infinity. The firms that cannot make profit in the production sites of the original problem will be better off by selecting the artificial site. The potential number of firms, N, plays the role of the given number of players n in the fixed entry model.

Suppose that firm r selects a production site i which satisfies conditions (i) and (ii). Then, firm r is viable and the payoff function can be represented as: $\pi_r(x_r, x_{-r})|_{x_r=i} \equiv \pi_{ir}(x_{-r})$. Since each market is served by only one production site from conditions (i) and (ii), the payoff of firm r is independent of strategies of the firms located at the other sites. Since the payoffs of the firms located at the same site are identical, the payoff of firm r can be represented as a function of the number of firms who select the same production site, $|Z_i|$; i.e., $\pi_{ir}(x_{-r}) = f_i(|Z_i|)$. From Eq. (6), $q_{ijr} = (\delta_{ij} - c_i)/(b_j(|Z_i| + 1))$ for all market j which satisfies the second condition, and thus $f_i(|Z_i|)$ is a non-increasing function. Therefore, this game is a special case of congestion games and there always exists a Nash equilibria.

(b) Proposition 2 and the existence of PNE of G_S imply the non-emptiness of *SS*. Let x be a location vector generated from the member of *SS* which includes the largest number of entrants among the members. Since $(a_j - v_{i^*} - t_{i^*} - c_{i^*})^2/4b_j > f_{i^*}$, at least one firm can exist in the industry. Suppose that there exists an entrant r who is not located at the site of the cheapest fixed cost. Then the entrant r will move to the site of the cheapest fixed cost without affecting the profit of the other firms, since $v_i + t_{ij} + c_i = v_{i'} + t_{i'j} + c_{i'}$ for all i, i' at every j. This procedure will be repeated until all entrants co-locate at the site with the cheapest fixed cost. The resulting location vector is a PNE, because no entrant can improve its profit and no outside firm can make profit by entering the industry, considering the fixed costs and identical total variable costs.

(c) Under the conditions of part (a), we showed that our game makes a congestion game. Milchtaich (1996a,b) addressed the relationship between simultaneous and sequential move game as follows:

> If G is a generic simultaneous move congestion game, then the backward induction profile of the corresponding sequential move congestion game is a Nash equilibrium (in pure strategies) of G.

This proves the first part of (c). Under the conditions of part (b), no firm has an incentive to move out of the production site of the cheapest fixed cost. Thus, $PNE(G_S) \subset PNE(G_T)$. □

Proof of Proposition 4. Let $x = (x_1, \dots x_{N^*})$ be a PNE of G_S. Suppose that there exists a firm r such that $\pi_r < \pi_{r+1}$. For a given $x_1 - x_{r-1}$ the game tree for backward induction is reduced to a two-person game (firm r and $r+1$), since they can foresee equilibria of the subgames starting from firm $(r+1)$'s decision node. Suppose that firm r can relocate its facility after firm $(r+1)$'s location decision, but does not want to. Then firm r should have selected firm $(r+1)$'s site in its initial decision. Therefore, the firm must relocate its facility, which is contradictory to $\mathrm{PNE}(G_S) \subset \mathrm{PNE}(G_T)$. $\square$

Algorithm for the second stage:

Step 1. For $j = 1$ to M,

(1) $k \leftarrow 0$.
(2) Sort production sites in descending order of δ_{ij}.
(3) $k \leftarrow k + 1$.
(4) If $\left(\sum_{l_j^k |Z_{l_j}|+1}\right)(\delta_{kj} - c_k) - \sum_{l_j}^{k} |Z_{l_j}|(\delta_{l_j j} - c_{l_j}) > 0$ and $\left(\sum_{l_j}^{k+1} |Z_{l_j}| + 1\right)(\delta_{k+1j} - c_{k+1}) - \sum_{l_j}^{k+1} |Z_{l_j}|(\delta_{l_j j} - c_{l_j}) \leqslant 0$, go to (5); otherwise go to (3).
(5) $l_j^* = \max\{l_j | l_j \leqslant k \text{ and } |Z_{l_j}| > 0\}$; Obtain q_{ijr}^*'s using Eq. (6).

Step 2. Obtain p and π_r's.

If we use a heap sort, sorting δ_{ij} in Step 1 takes $O(L \bullet \log L)$ and other small steps in Step 1 takes $O(L)$. Thus step 1 takes $O(L \bullet M \bullet \log L)$. The computing time of Step 2 is $O(L \bullet M)$. Therefore, the computational complexity of this algorithm is $O(L \bullet M \bullet \log L)$.

Algorithm to identify PNE (G_T) from SS:

Step 1: For $i = 1$ to L,

(1) $|Z_i'| \leftarrow |Z_i| - 1$;
(2) While $(k \neq i)$, $|Z_k' t| \leftarrow |Z_k| + 1$; if $\pi_k(|Z_1|, \dots, |Z_i'|, \dots, |Z_k'|, \dots, |Z_L|) > \pi_i(|Z_1|, \dots, |Z_L|)$, $\mathrm{SS} \neq \mathrm{PNE}(G_T)$ and stop; else, return to (2).

Step 2: $\mathrm{SS} = \mathrm{PNE}(G_T)$.

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