

A psychological approach to strategic thinking in games

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Psychologists have avoided using game theory because of its unrealistic assumptions on human cognitive ability, such as perfectly accurate forecasting, and its large reliance on equilibrium analysis to predict behavior in social interactions. Recent developments in behavioral game theory address these limitations by allowing for bounded and heterogeneous thinking, recognizing limitations on people's forecasting abilities, while keeping models as generally applicable as those using equilibrium analysis. One such psychological approach is cognitive hierarchy (CH) modeling, in which players reason accurately only about those who think less. CH predicts non-equilibrium behaviors that have been observed in more than 100 laboratory experiments and several field settings, and has process implications that have been tested with eye-tracking and data from brain imaging.

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Introduction

Game theory is a formal approach that has proved useful in economics and political science, as well as certain areas of computer science, sociology and biology. However, it has gotten little traction in cognitive and social psychology because of its strong assumptions on human behavior. Recent developments address these limitations, and in this paper, we will describe one important development in understanding strategic thinking and aspects of sociality.

Game theory mathematically describes social interactions in which the actions of one player influence what happens to another. A description of game or strategic interaction consists of players, their strategies, the information they have, the order of their choices, and the utility they attach to each outcome. Outcomes include tangibles (money earned in an experiment or poker winnings) and intangibles (emotional satisfaction from enforcing norms or being ahead of others).

Until the 1990s, most game theorists relied on equilibrium analysis to predict strategies. Players are considered to be in equilibrium when they correctly forecast what others will do and pick a utility-maximizing strategy. For example, always playing 'rock' in rock-paper-scissors isn't an equilibrium strategy. Only play 'rock' if you think your opponent will play 'scissors,' but your opponent won't do that if they think you're going to play 'rock'; instead, they will play 'paper'. Therefore, the equilibrium strategy is to randomly play each hand 1/3 of the time.

An equilibrium can be a useful prediction of where a social system may end up because theory and evidence suggest that adaptive learning and evolutionary selection lead to equilibration [1,2,3*]. However, because of cognitive limitations, it is psychologically implausible for people to derive equilibrium strategies purely from thinking. Therefore, equilibrium analysis poorly predicts outcomes when players encounter new games or there is a shift such as a policy or technology change.

In the mid-1990s, game theory was extended to include behavioral models of strategic thinking, called cognitive hierarchy (CH) or level-k models [4,5**,6]. We focus on CH models, which have been applied to more than 100 experimental games and field settings, and whose process implications have been tested with eye-tracking and data from functional magnetic resonance imaging. This shift centralizes psychology, making cognitive representations of game structures [7], strategy categorization, and cross-game learning [8] researchable questions, unlike in standard game theory.

The CH approach

Behavioral game theory not only extends equilibrium analysis, it also generates psychologically plausible predictions by relaxing two central assumptions: (1) players always choose a utility-maximizing strategy, and (2) players correctly forecast what other players will do.

In an approach called softmax, pioneered in mathematical psychology, (1) is relaxed by allowing players to respond

stochastically. When (2) is maintained, a quantal response equilibrium results, which makes precise predictions and explains many observed deviations from equilibrium predictions [9,10,11**].

The CH approach keeps (1) but relaxes (2), allowing variation in guessing, because of which systematic, non-equilibrium behavior can occur. Precision is maintained by a strategic thinking hierarchy where higher-level thinkers understand what lower-level thinkers are likely to do (see Box 1). Level-1 thinkers believe they are only facing level-0 opponents; level-2 thinkers believe they are facing level-0 and level-1 opponents, and so on. Once the level-0 behavior and the proportion of players at levels k ($f(k)$) are specified, the distribution of predicted strategy frequencies can be evaluated. Level-0 thinkers choose a salient strategy such as an auspicious number or choose all strategies equally if none are salient. Completing the specification, the Poisson distribution with one parameter τ parsimoniously captures the proportion of levels k well.

Box 1 Mathematical details

There are five elements to any CH or level k model:

1. Distribution of the frequency of level thinkers, $f(k)$
2. Actions of level-0 thinkers, who act heuristically and without beliefs about other players
3. Beliefs of level- k thinkers (where $k = 1, 2, \dots$) about other players' choices
4. Assessment of expected payoffs for each level- k based on (3)
5. Players choosing the payoff-maximization strategy

The typical approach is to make precise assumptions about (1) to (5), leaving parametric flexibility in the distribution $f(k)$, like $k - 1$ degrees of freedom (frequencies sum to 1), or a particular shape of $f(k)$ depending on one or two parameters. Then we see how well the model fits experimental data from different games and what the best-fitting parameter values are. If the model fails badly, it can be extended and improved. The hope is that a common specification (similar $f(k)$ and level-0 choices) explains behavior in games with different payoff structures, strategies, and numbers of players.

In [5**], the distribution of level- k types is assumed to follow a Poisson distribution $f(k) = \exp(-\tau)\tau^k/k!$, which has mean and variance τ . This distribution is implied by the axiom $\frac{f(k)}{f(k-1)} \propto \frac{1}{k}$, so if the graduation rate of level $k - 1$ players to level k drops proportionally with $1/k$, then $f(k)$ has a Poisson distribution. This distribution is parsimonious (only one parameter, τ) and $f(k)$ drops off rapidly as k increases because of the $k!$ factorial in the denominator, approximating the increasing difficulty of hierarchical reasoning. For example, if $\tau = 1.5$, then less than 2% of players are expected to do five or more thinking steps.

Level- k is an alternative approach [6], which assumes that players think others are all precisely one level below. The level- k and Poisson CH models do not differ for level-0 and level-1, and their differences at higher-levels are typically not empirically large. A Bayesian meta-analysis [11**] indicates CH fits a little better than level- k , provided a spike of level-0 players is added to the Poisson distribution. An important extension to sequential games has also been made [12].

Besides being behaviorally more plausible, CH has another advantage. Consider a stag hunt game where 2 hunters must decide whether to hunt for a hare individually or for a stag together [13]. If both choose stag and do indeed persevere, each gets X units payoff; if either one switches to hare, the other gets 0. Hunting for hare always pay 1 unit regardless of what the other does. Both choosing stag and both choosing hare are two equilibria. Standard game theory does not provide clear guidance which is more likely. CH does. If X is 2 or more, CH predicts stag as it is more profitable to levels 1 and above; if X is less than 2, hare is a more profitable choice.

Examples of equilibrium, CH predictions and data

Consider three games

p-beauty contest (PBC)

Simultaneously and without communicating, each player chooses a number from 0 to 100. The player whose number is closest to an announced value p , multiplied by the average of the chosen numbers, wins [14,15].

Suppose p is $2/3$. If level-0 thinkers choose all numbers from 0 to 100 equally, a level-1 thinker will think the average is 50 and choose 33. This is reasonable, but isn't equilibrium since choosing 33 while anticipating others choosing 50 means that guessing 50 is incorrect. Level-2 thinkers think that the average of level-0 and level-1 players is a number between 33 and 50, and will themselves pick a number between 22 and 33. The unique equilibrium predicts everyone will expect and choose 0.

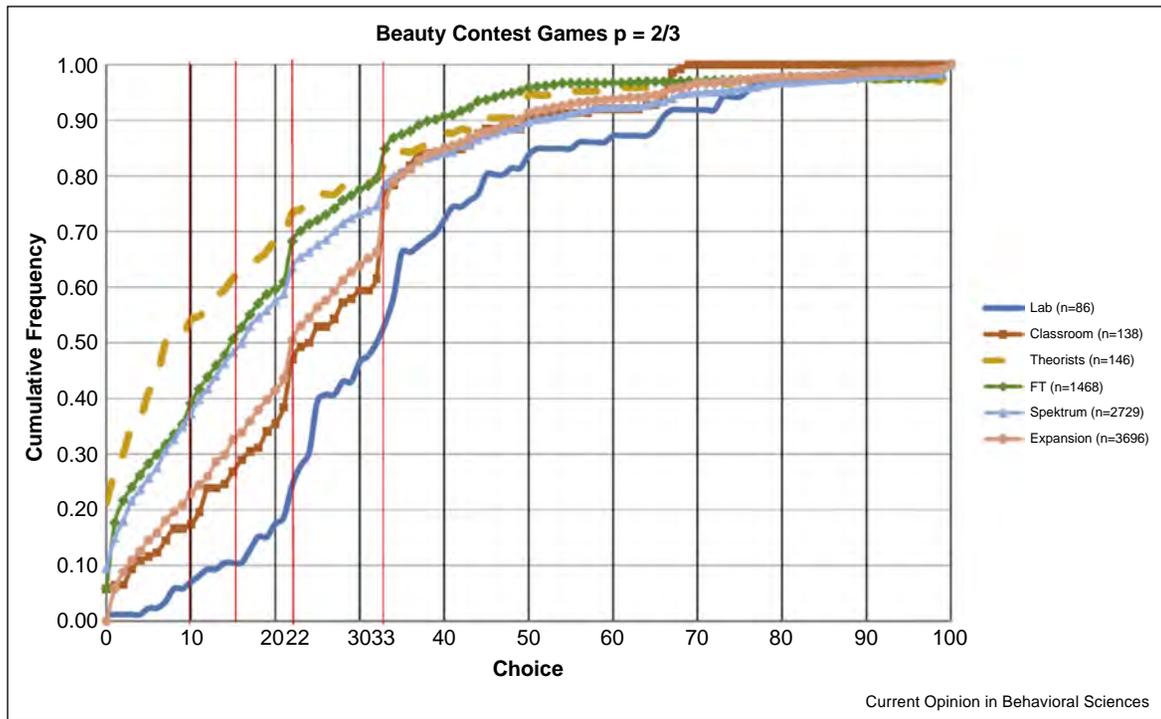
Figure 1 shows data from newspaper and magazine contests where p is $2/3$. There is evidence of small spikes in numbers corresponding to $50p$, $50p^2$, and so on [16], showing that people exhibit different levels of strategic thinking, as predicted by CH but not by equilibrium analysis.

LUPI

The goal is to pick the lowest unique positive integer (LUPI) from 1 to 99,999 [17]. To win, a player must balance two goals – pick a low, unique number – which requires strategic thinking about what other players are likely to choose.

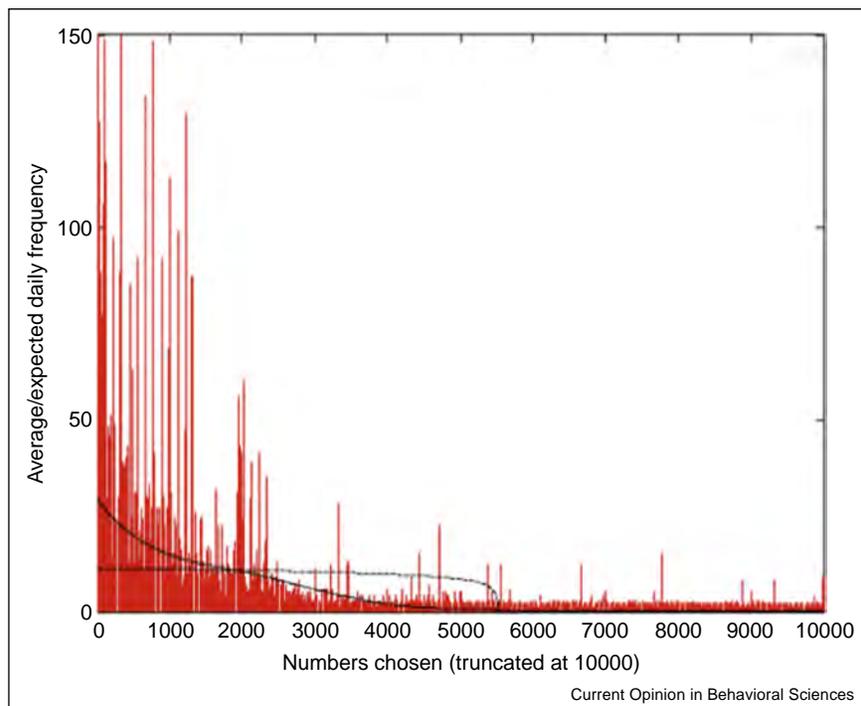
CH predicts that low-level thinkers will choose low numbers because they don't anticipate others choosing the same low numbers. For example, the most common choice was 1, which is a low number but not a unique one. Equilibrium analysis assumes that all players guess the distribution of numbers. In this case, the equilibrium is mixed since probability is spread throughout the numbers. This distribution puts the highest weight on 1, slightly less on 2, and so on until a sharp drop at 5513 (Figure 2, dotted line). This precise, non-obvious prediction is mathematical but comes out of thin air with no free behavioral parameters.

Figure 1



Several cumulative distribution functions (CDFs) from pBC Game, played in labs and in the field; $p = 2/3$. Vertical lines at 33 and 22 show jumps in the CDF corresponding to level 1 and 2 player choices, especially in lab and classroom data.

Figure 2



Histogram from LUPI game in Sweden.

Figure 2 shows that players chose too many low numbers and too few numbers between 2500 and 5000 than equilibrium prediction. The CH model can fit this pattern using an average thinking level $\tau = 1.80$, with τ derived from lab experiments.

Entry

Players must choose whether or not to enter a competitive market. A market with demand D is announced to $n = 12$ players [5,16]. Players who don't enter earn \$0.50. Players who enter earn \$1 if total entrants are D or less or \$0 if there are more than D entrants.

Equilibrium analysis predicts that D players will choose to enter, but CH predicts that (ignoring the value of D) if level-0 thinkers enter half the time, level-1 players will only stay out if $D < n/2$. Summing across levels gives a CH distribution that looks similar to the equilibrium, except that there is too much entry at low D values and too little entry at high D values.

Experimental data (Figure 3) matches the CH prediction closely, with $\tau = 1.50$. In fact, after seeing his data, Kahneman [18] was amazed at how closely total entry tracked D , writing 'To a psychologist, it looked like magic.' CH explains this by predicting that higher-level thinkers fill in gaps between D and the expected number of entrants. When they expect too little entry by lower-level thinkers, they themselves enter, bringing the total close to D .

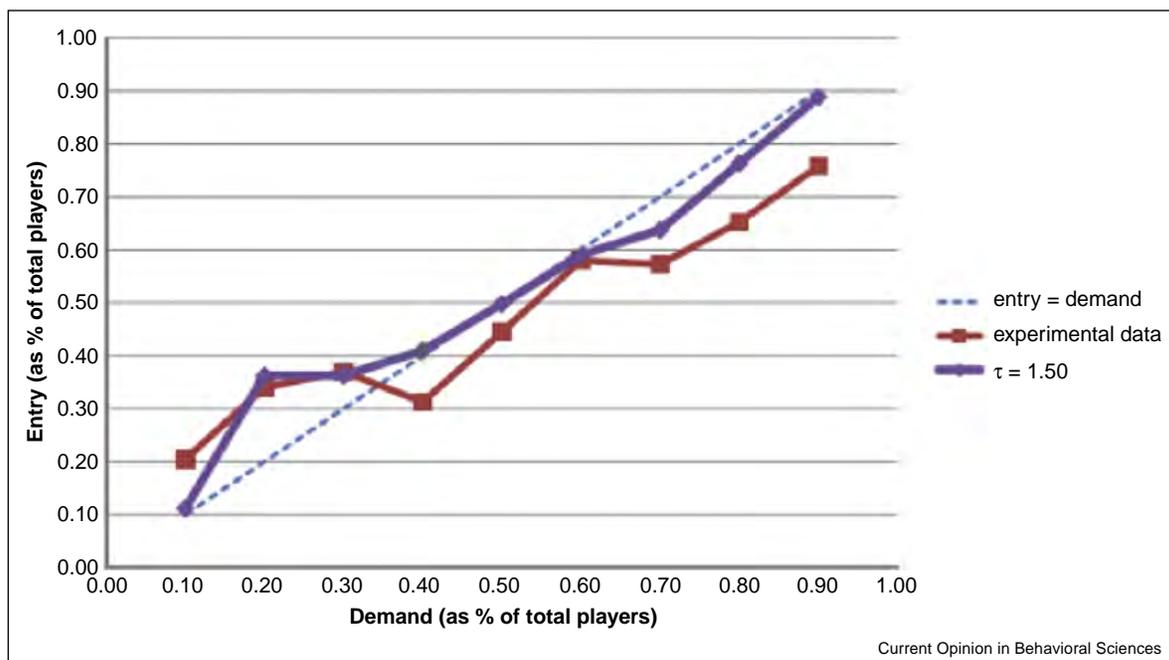
The contrast between Figures 1–3 makes a profound point about rationality and social incentives. In the PBC game, higher level thinkers anticipate players deviating from the equilibrium of 0, and themselves deviate to approach the average, engaging in strategic complementarity. However, in the LUPI and Entry games, when players deviate from the equilibrium by choosing low numbers or entering too often, higher level thinkers pick unique numbers or avoid over-entry, engaging in strategic substitution. This explains why behavior is 'magically' close to equilibrium the first time people play. Furthermore, whether a game exhibits complementarity (non-equilibrium) or substitution (equilibrium) is mathematical and structural, allowing predictions on whether aggregate outcomes will be close to or far from equilibrium [19,20**].

Further experimental applications and field data

Without formal analysis of CH-type models, cognitive scientists have studied reasoning steps using a sequential-move matrix paradigm [21**,22,23] and have even discussed them in analyzing fiction [24,25]. Formal analyses have been applied in common-interest coordination games [26,27], auctions [28], hide-and-seek games [29], and the emergence of dominant platforms in 2-sided markets [30].

In games such as poker, each player has private information (their cards) that other players know about but don't

Figure 3



Comparison of experimental and CH models with data from the entry game. The CH model with $\tau = 1.50$ (triangle) fits the entry data (squares) better than the Nash equilibrium prediction (dotted line).

know the value of. CH models predict that level-0 and level-1 thinkers won't understand how this information can guide a player. This leads to interesting phenomena that are not possible in equilibrium analysis, such as dishonest sellers exploiting gullible buyers [31**,32]; the 'winner's curse,' where bidders can only guess the value of an object (e.g., oil lease auctions), perhaps too optimistically [28,33]; and zero-sum speculation in which two people should never agree to bet each other, but often do [34,35].

CH models have also been applied in field settings. These applications are challenging because they require simplifying complicated choices and uncertain payoffs into a form that can be analyzed mathematically. It then becomes difficult to tell if the poor explanatory power of a model is due to simplification or because the model is wrong. Fortunately, empirical applications conducted so far indicate that limited strategic thinking helps explain choices when compared to an equilibrium reasoning benchmark. Besides the LUPI lottery [17], CH has been used to analyze timber auctions [36], corporate manager strategies [37,38] and the reactions of moviegoers to review and non-review information [39,40].

Evidence from measuring attention and brain activity

Cognitive process theories like CH can also be tested using data such as verbal reports [41], response times [34] and visual fixations. Beginning twenty years ago, studies using mouse-based information acquisition and eye-tracking have shown that subjects are limited in their thinking by degrees, consistent with CH [42*,43**,44]. The hierarchical structure is also justified by the likely association and benefit between higher level thinking and better working memory [45].

Numerical measures of strategic thinking such as τ can also be used to identify brain circuits that encode these measures. This approach has been successful in studying non-strategic decisions [46,47] but has been applied infrequently to games [48**,49,50].

Conclusions

Models in which people do levels of strategic thinking are a tidy, cognitively-plausible way to understand limits on thinking and provide measurement tools and codification on how treatments and differences influence strategic thinking. This essay shows a mathematical way to model hierarchical levels of thinking, explaining behavior in a number of experimental and field settings. Furthermore, thinking steps can be empirically associated with patterns of visual fixation on information and neural processes.

Cognitive scientists can help develop two frontiers in this approach. The first is to derive the thinking-level distribution $f(k)$ endogenously from game features and player

traits. One idea is that players determine whether another level of thinking will increase the payoff beyond the cost of thinking, before performing it. This approach depends on correctly specifying how the determination works and what the cost of thinking means biologically.

The second is to specify, ideally from game structure and plausible perception, what level-0 thinkers do since their behavior crucially influences the entire hierarchy. Social psychologists could help develop a model with variables like a predictive concept of salience (perhaps cultural) and personal experience. For example, a recent study suggests that level-0 play is influenced by the fairness and extremity of payoff distributions [51].

Conflict of interest statement

Nothing declared.

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